

Gassmann's fluid substitution paradox on carbonates: seismic and ultrasonic frequencies

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Summary

A set of carbonate plugs of different porosity, permeability, mineralogy and texture are measured at seismic and ultrasonic frequencies in the laboratory. For this set we show that within the uncertainty of the measured shear and bulk moduli (8% in average), Gassmann correctly predicts the observed values for brine and butane saturation. This is observed for both seismic and ultrasonic frequencies which involve independent experimental setups and procedures. Moduli dispersion over the range of acquired frequencies exists for mostly for brine saturation, and is less for dry samples. This effect should also be considered when studying if the predicted data matches the observations. However, the current data set is not statistically representative, and the validity of Gassmann's theory on carbonate samples should still be applied with care. More samples are being analyzed to have a better representation of carbonate rock properties.

Introduction

Experimental data on carbonate reservoir rocks have not been as thoroughly studied as clastic sedimentary reservoir rocks. Domenico (1984), Rafavich et al. (1984), Anselmetti and Eberli (1997), Assefa et al. (2003), Prasad and Nur (2003) and Baechle et al. (2004) among others have described the ultrasonic velocity response of carbonate rocks to porosity, permeability, texture, fluids and pressure. Velocities, in general, decrease as the porosity increases. Texture and pore shape controls velocities, pressure and fluid effects. These dependencies are observed in our samples, but the focus of this work is to understand the applicability of Gassmann (1951) fluid substitution theory at seismic and ultrasonic frequencies in the context of uncertainty in the experimental data.

Two main requirements of Gassmann's equations are usually violated when studying carbonates. First, the interconnection of pores is usually not true for carbonate samples. Second, Gassmann's formulation requires that fluid saturation affects only the bulk modulus and *not* shear modulus. In carbonates, however, this is not true. Several studies have shown that the shear modulus changes between 5 and 20% from dry to brine saturated carbonates (Japsen et al., 2002, Assefa et al. 2003, Bachle et al 2003). Not only does the saturating fluid have a significant effect on the shear moduli, but Vo-Thanh (1995) showed that the percent of saturation changes the shear modulus by 11% from a dry to a completely water saturated limestone sample. However, although at least two of the conditions for Gassmann are violated, most carbonate samples at high pressures (reservoir pressures) show that the calculated P-wave velocities seem to predict ultrasonic laboratory

measured P-wave velocities within about 5% (Wang, 2000, Japsen et al., 2002, Assefa et al., 2003 and Baechle et al., 2003). In some of these studies, subsets of analyzed samples show up to 20% difference between measured and predicted. At lower pressures, where open induced cracks/fractures are present, the difference between measured and predicted by Gassmann saturated velocities increases.

This work is different from previous research because it not only analyzes carbonate samples at ultrasonic frequencies, but includes the seismic frequency range as well. This larger frequency data set provides estimates of the moduli (velocity) dispersion. Gassmann's formulations predict our experimental data within the uncertainty of the observations and the moduli dispersion effects.

Data, processing and uncertainty

The available data consists of 10 carbonates over a large range of porosity (5-35%), permeability (0.001-800 mD), mineralogy (dolomite and limestone), and textures/pores (for example: micritic, fossils, vugs). The samples were either almost pure calcite or dolomite (95%) with less than 1% clays. Anisotropic or largely vuggy samples were avoided. So far 4 of the 10 samples have been measured at low (seismic: 3-3000 Hz) and ultrasonic frequencies (~1MHz). The samples are cylindrical, 1.5" in diameter and 1-2" in length. The low frequency measurements follow the stress-strain methodology described by Spencer (1981) and Batzle et al. (2001). The ultrasonic data is acquired by propagating a wave (pulse) through the sample. The low frequency data has shown larger values of Poisson's ratio than expected, possibly due to large deformation of the sample in the horizontal direction related to the end effects present in our stress-strain system. Figure 1 shows Poisson's ratios computed from fits to ultrasonic P- and S-wave vs. porosity from brine saturated carbonates. These fits are from Domenico (1984), Anselmetti and Eberli (1997), Mavko (1997), Assefa et al. (2003) and Han (2003). The black dots are our ultrasonic measured samples. Because our data points agree with observations from previous studies we use these four measured ultrasonic Poisson's ratios to compute bulk and shear moduli (later velocities) for the low frequency data. This is possible because no change of Poisson's ratio with frequency is observed. We are developing a finite element code to understand the nature of these anomalous Poisson values at low frequencies.

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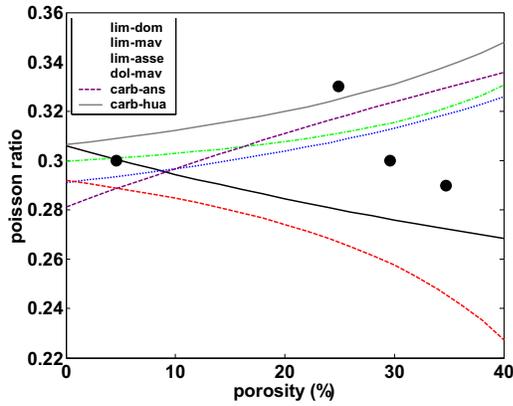


Figure 1: Poisson's ratio, compiled from different studies, computed from fits to brine saturated carbonate samples from ultrasonic velocities (lim=limestones, dol=dolomites). Black dots are our ultrasonic data.

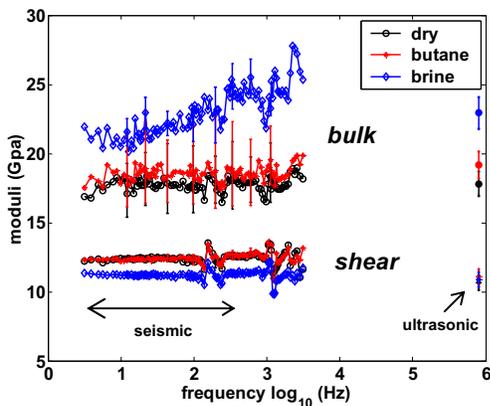


Figure 2: Dispersion effect on the moduli for different fluid saturations (dolomite).

Uncertainty on the low frequency data relates to the response (deformation) of the different strain gages on the sample. We usually work with an average value, but heterogeneity in the sample might affect the strain estimates. Figure 2 shows the error bars for the moduli for a dolomite sample. The error in the moduli is 8% in average. For ultrasonic data, error in the first arrival time of the wave and the precision of the instrument to measure sample length produce an error in the velocities of about 5%.

Finally, for the low frequency experiment we have a range of frequencies, but for this analysis we restrict ourselves to one representative low frequency. First, we apply a least-squares fit to the frequency versus moduli for each sample and saturation (Fig. 3). This procedure is only for smoothing purposes. In subsequent analysis, we use the smoothed values at 3, 100 and 3000 Hz frequencies, but

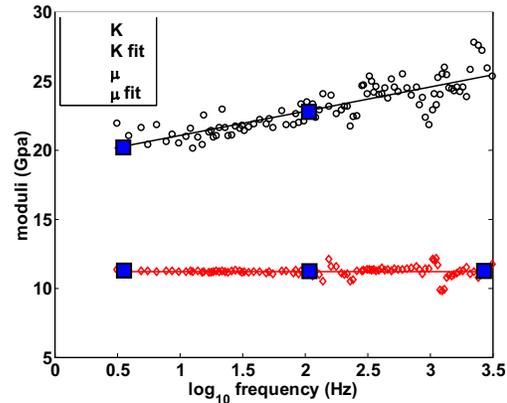


Figure 3: Bulk (K) and shear (μ) moduli for sample B. The line fit is used to smooth the data. All the other samples and saturations show similar results. Squares represent moduli calculated at 3, 100 and 3000 Hz.

mainly at 100Hz. These three distinct frequency values give us estimates of dispersion on the bulk and shear moduli.

Results

The results that follow include data for 4 carbonate samples. Here we analyze data at a differential pressure of 3000 psi although the samples were measured from 500 to 4500 psi with intervals of 500 psi. The reservoir pressure is about 4500 psi. The saturating fluids used were butane (a light hydrocarbon) and brine (200,000 ppm salinity).

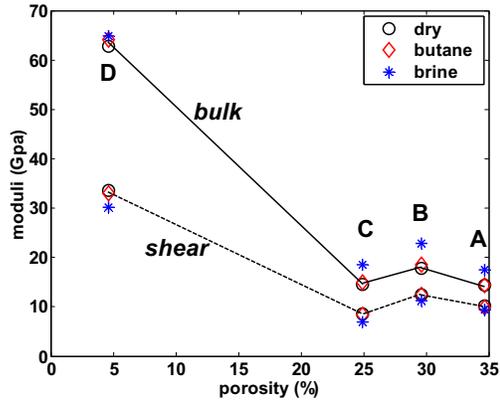


Figure 4: Bulk and shear moduli at 100Hz. The presence of fluid stiffens the bulk modulus while it weakens the shear modulus.

As expected, when samples are saturated with brine, the bulk modulus increases compared to the dry modulus. In contrast, the shear modulus decreases with brine saturation (Fig. 4). This is consistent with observations from studies mentioned previously. Butane, being a non-

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polar fluid and more compressible than brine, does not show a large moduli difference compared to dry.

Figure 5 shows the shear modulus change from dry to brine saturation. In average, the percent difference between these saturation conditions varies among samples by 13% for low frequency and 9% for ultrasonic. This is a significant change and violates Gassmann’s requirement that the shear modulus does not change with saturation. The boxes in Fig. 5 represent the dispersion effect on the shear moduli (3 and 3000 Hz). Note that the dispersion effect is mostly present on the brine shear moduli and it is in average 6%.

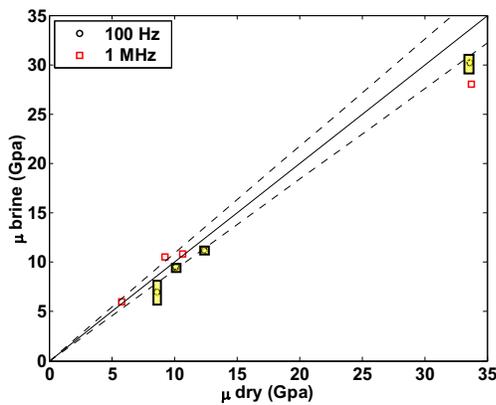


Figure 5: Shear moduli change with brine saturation. Squared are ultrasonic data and circles are at 100Hz. The boxes represent the dispersion boundaries (3 and 3000 Hz) for the shear moduli. Solid line is for $\mu_{dry} = \mu_{brine}$; dashed lines give 8% deviations from this.

Gassmann substitution

Gassmann estimates the fluid-saturated bulk modulus of a rock from the dry rock, mineral, and fluid moduli and porosity. This estimate is compared to the measured saturated bulk modulus at 100 Hz and at ultrasonic frequencies (Fig. 6). Here the solid line means that the fluid-substituted and measured moduli are equal, and the dashed lines represent an 8% deviation. It can be seen that most estimates fall within the dashed lines. This choice of 8% deviation is based on the average error in our data (see section on Data, processing and uncertainty). Any estimates that fall within these lines should be considered to have no significant variation from the measured values. Thus, Gassmann is correctly predicting the observed brine saturated moduli in this particular case. The difference between measured and predicted butane saturated bulk moduli is also less than 8%.

Figure 7 shows brine saturated P-wave velocities for two scenarios. The Gassmann fluid-substitution for bulk modulus is made using dry and brine saturated shear moduli. For both cases, the estimates lie within an 8% error constrain, meaning that for this set of samples, at seismic

and at ultrasonic frequencies, Gassmann substitution agrees with the observed values in our experiments. No significant differences in the velocity estimates are present if the dry or brine shear moduli are used when considering the data confident interval. Similar agreement between predicted and measured data is observed for data at 3Hz and 3000Hz.

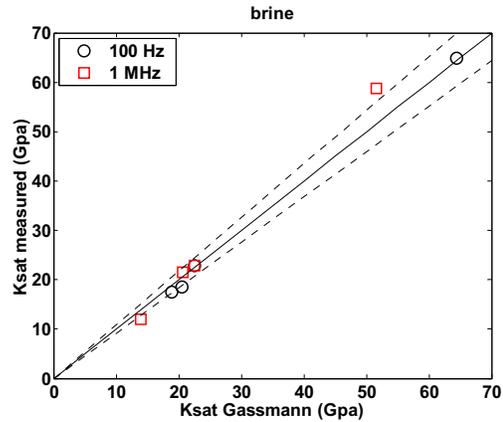


Figure 6: Measured brine saturated bulk moduli estimated by Gassmann substitution and at low and ultrasonic frequencies. Solid line represent moduli are equal, dashed lines are 8% deviations from it.

As the dispersion effect on the moduli is in average 6% its contribution lies within the 8% confidence interval for this data. If the measurement error is lower than the dispersion effect, or the later is larger than the experimental error, moduli dispersion should be accounted when interpreting the applicability of Gassmann’s formulation if available.

Surprisingly, although two of Gassmann’s requirements are violated, the equations still worked on this set of samples. This analysis was done for a differential pressure of 3000 psi. Lower pressures show an increasing difference between measured and predicted bulk moduli (P-wave velocity). The results shown here are below the reservoir pressure. It is expected that at higher pressures the data and predictions would have a better match.

Although the results at the moment are only for 4 samples they have been estimated by two independent methods, producing similar results. Also they represent different mineralogies, porosities, permeabilities and textures.

We need to be careful when we generalize the applicability of Gassmann substitution. On a larger data set, some samples have shown up to 20% mismatch between measured and predicted velocity values (Wang, 2000, Baechle et al., 2003). Sample characteristics (porosity, texture, permeability, etc.) might help us to understand this large variation. Also, uncertainty in the estimates might aid

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to interpret the applicability ranges of the predicted and estimated values for those samples.

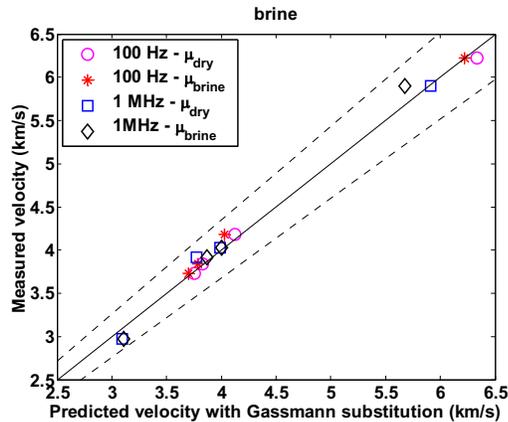


Figure 7: Brine saturated P-wave velocity estimated by Gassmann substitution and measured at low and ultrasonic frequencies. Solid line represent equal velocities, dashed lines are 8% deviations from it.

Conclusions

Uncertainty analysis is important in any area in geophysics, especially where conclusions are based on quantitative results. Uncertainty should be accounted for when estimating velocities or moduli from laboratory experiments. Here we study carbonate samples in experimental settings for seismic and ultrasonic frequencies. Gassmann's theory is applicable to this small set of carbonate samples. Previous studies have shown that Gassmann theory does not predict saturated bulk moduli or velocities because the difference between estimated and measured P-wave velocity is 5-10%. We observe similar mismatch, but if experimental uncertainty is included, the predicted values fall within the confidence interval of the experimental data. Dispersion effects are observed mostly for brine saturated sample, and less for dry measurements. The variation of moduli due to the dispersion is in average 6%, meaning a range, rather than a point when cross-plotting moduli (for example dry vs. brine, measured vs. predicted). The error between Gassmann predicted and measured data increases with decreasing differential pressure. In this study, we observe good correlation for a pressure 30% lower than at the reservoir.

This is work in progress and the rest of the data samples will be measured to provide a better statistical representation of the carbonates supporting our conclusions.

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EDITED REFERENCES

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