Effective pressure or what is the effect of pressure?

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Using time-lapse seismics as a reservoir-monitoring tool, geophysics can help distinguish different reservoir production scenarios. For example, Eiken et al. (2000) successfully detected fluid-saturation changes after CO_2 injection using time-lapse seismics at Sleipner Field. Over the cycle of a reservoir life, oil saturation usually decreases, reservoir pressure declines, and gas breakout may occur. These changes cause rock property changes that are detectible in time-lapse seismics. Therefore, it is important to understand the effects of pressure and saturation changes on rock properties. While the effects of saturation changes are often well described by Gassmann (1951), Brown and Korringa (1975), and Mavko (1975), the effects of pressure changes are less understood. Here we focus on understanding the effects of fluid pressure on velocities.

Reservoir-pressure changes depend on production schemes. For example, under weak acquifer-drive conditions, the pore pressure tends to decrease during production, and therefore, the differential pressure (the difference between confining and pore pressure) increases. In contrast, in the case of fluid injection, the pore pressure can increase near injection wells.

Both theoretical and empirical methods are used to understand the effects of pressure changes on rock properties. The Hertz-Mindlin model uses spheres to predict theoretically the effects of pressure changes on the rock stiffness. Using their approach, the increase of rock stiffness and rigidity with the differential pressure is caused by the increase of contact area between spheres. Eberhart-Phillips et al. (1989) show empirically that rock velocities increase exponentially with differential pressure. Using local calibration from core measurements, coefficients for these equations can be found and used to predict pressure changes due to production.

However, several case studies show that our understanding of how rocks respond to pressure changes is still limited. MacBeth et al. (2004) show that the predicted time-lapse amplitude changes are much larger than the measured amplitude changes for a field in the UK's Southern Gas Basin. Tondel and Eiken (2005) also show that for Troll Field, the seismic 4D response is much smaller than the predicted response.

Several causes are given to explain these discrepancies between the predicted and observed time-lapse responses. One of the most plausible reasons is the misunderstanding of the concepts of differential and effective pressure. To be more accurate, we should consider stress rather than just pressure; however, for our discussion here, we restrict ourselves to the simpler isotropic case.

Differential versus effective pressure. The differential pressure (P_d) is the difference between confining pressure (P_c) and pore pressure (P_p),

$$P_d = P_c - P_p \tag{1}$$

For soils and unconsolidated rocks, Terzaghi (1936) shows that P_d governs the pressure dependency of soil and soft rock deformation. For consolidated rocks, however, Biot

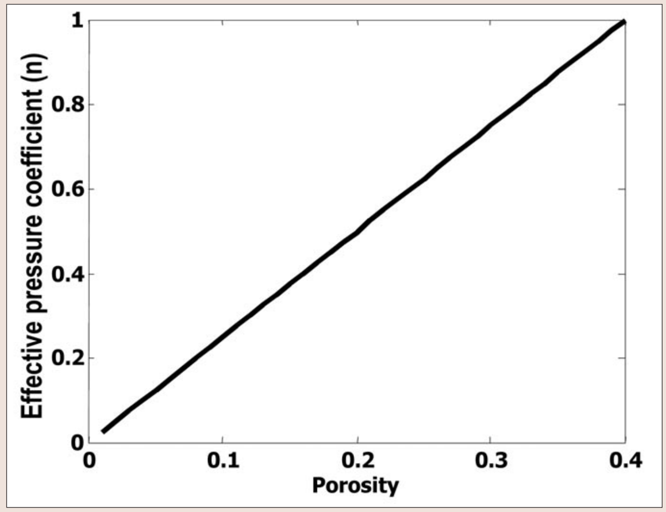

Figure 1. The effective pressure coefficient for bulk modulus as function of porosity. The critical porosity concept is applied to calculate the dry bulk modulus, which is then applied through the Biot-Willis theory.

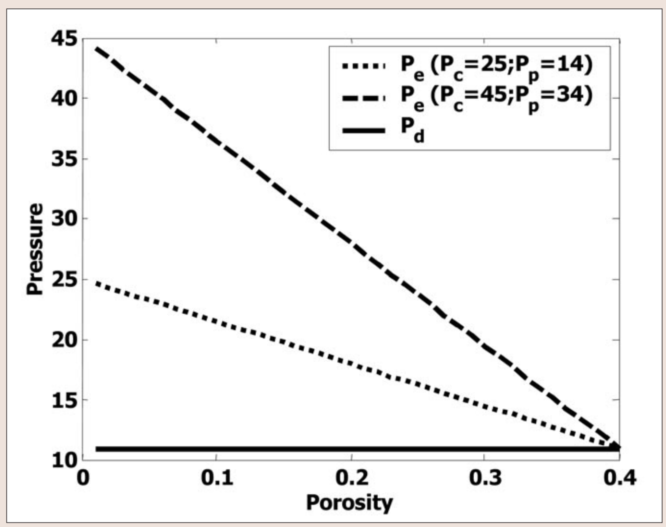

Figure 2. Effective pressure as function of the porosity. For two in-situ conditions (P_c=25 MPa, P_p=14 MPa; P_c=45 MPa, P_p=34 MPa) the differential pressure is constant (P_d=11 MPa) as function of porosity; with a decrease of porosity, the effective pressure increases depending on P_c and P_p.

and Willis (1957) show that rock deformation is not solely dependent on P_d, but rather, on effective pressure,

$$P_e = P_c - nP_p \tag{2}$$

The effective pressure differs from the differential pressure by the so-called effective-pressure coefficient, n. Their theoretical derivation relates the coefficient to the dry modulus, K_{dry}, and mineral modulus, K_m, of the rock:

$$n = 1 - \frac{K_{dry}}{K_m} \tag{3}$$

This relation is derived from static deformations of the rock frame and may differ from empirical relations for dynamic properties. As an upper bound for these empirical relations, the critical porosity model (Nur et al., 1995) can be applied to obtain the effective-pressure coefficient:

$$n = \frac{\phi}{\phi_{cr}} \tag{4}$$

where the critical porosity ϕ_{cr} is 0.4 for clastic rocks. Figure 1 and Figure 2 show the results of this model and its implication for an effective-pressure coefficient. As indicated in Figure 1 at high porosities, where the dry bulk modulus is very small, the effective-pressure coefficient is close to one. Conversely, if the porosities are very small, the effective-pressure coefficient is very small, approaching zero. Effective pressures (equation 2) for two different pressure conditions are shown in Figure 2. Differential pressure remains the same for both scenarios: P_c=45 MPa, P_p=34 MPa, P_d=11 MPa and P_c=25 MPa, P_p=14 MPa, P_d=11 MPa. However, the effective pressure can be very different, ranging from 25 MPa to 45 MPa at very low porosities (n=0), in contrast, at very high porosities the effective pressures converge to the differential pressure (n=1).

Todd and Simmons (1972) derive an empirical equation, yet more general, for effective-pressure coefficients by taking the ratio of the change of any property Q at constant differential pressure (P_d=const) over the change at constant pore pressure (P_p=const):

$$n_Q = 1 - \frac{\left.\partial Q/\partial P_p\right|_{P_d}}{\left.\partial Q/\partial P_d\right|_{P_p}} \tag{5}$$

As illustrated in Figure 3, the denominator in equation 5 can be obtained from the tangent of the constant pore pressure curve, while the numerator is the slope of the curve for constant differential pressure. Zoback et al. (1975) measured the effective-pressure coefficient for permeabilities, whereas Prasad and Manghnani (1997) measured the coefficient for compressional velocity and quality factor. The effective-pressure coefficient for rock properties, such as velocity and permeability, depends on the deformation behavior of rocks. Because deformation affects permeability and velocity differently, the effective-pressure coefficient is different for each property.

For rocks with high porosity the value of the coefficient is very close to one, which means that changes in confining and pore pressure have the same effect on permeability and velocities. In contrast, rocks with low porosity are influenced more by changes in confining pressure than by changes in pore pressure. Therefore the effective-pressure coefficient is smaller than one. When the effective-pressure coefficient approaches zero, the pore pressure has no influence on deformation, velocity, or permeability.

When a rock is subjected to constant differential pressure, pore pressure has to be changed accordingly; therefore, fluid compressibility and density change. Figure 4 shows schematically how the increase of velocity at constant differential pressure is caused by the increase in fluid stiffness and the increase in rock frame stiffness. As an example, Figure 5 shows the change in brine density and modulus as a function of pore pressure. As expected, the density change is nominal, from 1.03 g/cm^3 to 1.10 g/cm^3, a total of 7%. The fluid modulus, however, changes from 2.5 GPa to 3.6 GPa (44%) as the pore pressure increases from 7 MPa to 170 MPa. To remove this fluid contribution to the increased rock stiffness, we use Gassmann's equation to calculate the

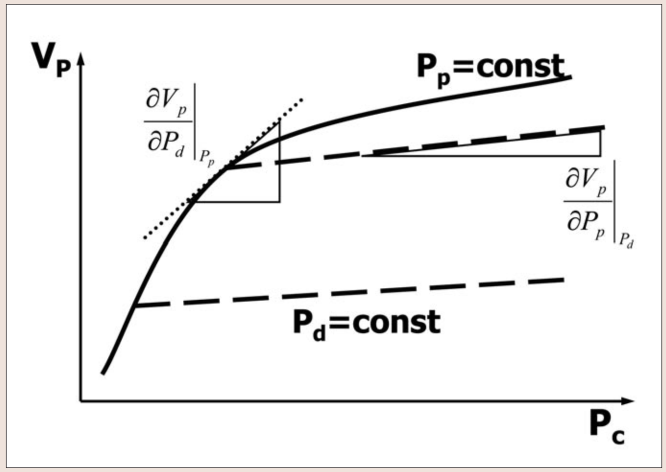

Figure 3. Schematic behavior of compressional wave velocity (V_P). The two derived quantities for constant differential and constant pore pressure are indicated.

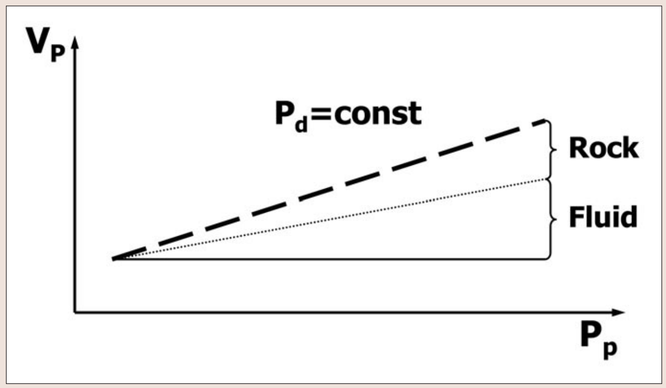

Figure 4. Compressional wave velocity (V_P) as function of pore pressure. The contributions from changing fluid properties and rock properties are indicated.

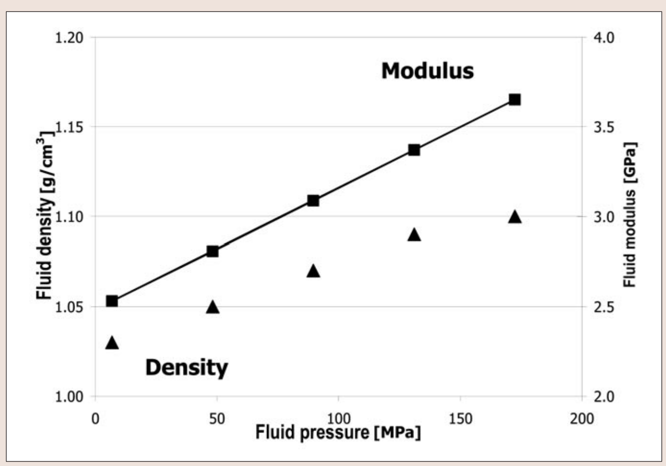

Figure 5. Changes in brine density and modulus as function of the pore pressure (triangles indicate density - right axis, squares indicate modulus - left axis).

stiffness of a rock frame under a constant pore pressure. In this study, all the brine-saturated measurements have been normalized to fluid properties at constant pore pressure of P_p=3.9 MPa (1000 psi).

Given the importance of the effective-pressure coefficient at low porosities, measurements of a Cretaceous sandstone with a porosity of 13% are presented in this analysis. At first, the normalization of the brine properties is applied; the

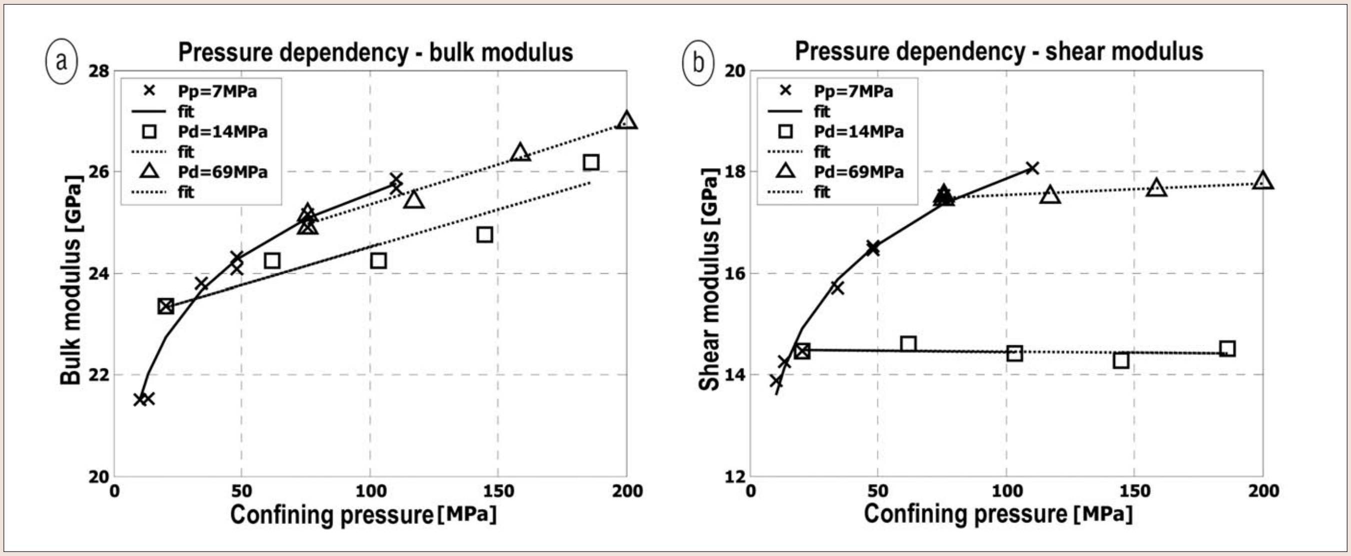

Figure 6. *Pressure dependency for normalized saturated bulk (a) and shear modulus (b) as function of confining pressure, for constant pore pressure (P_p=7 MPa), and constant differential pressure (P_d=14 MPa and 69 MPa). All brine-saturated data have been normalized to the properties of brine at P_p=3.9 MPa (1000 psi).*

resulting bulk and shear moduli are plotted in Figure 6 as functions of confining pressure and at constant pore pressure P_p=7 MPa and at two constant differential pressures (P_d=14 MPa and P_d=69 MPa). If the effective-pressure coefficient would equal one, the bulk and shear moduli for constant differential pressure would not change as a function of increasing confining pressure. For the shear modulus, this is essentially the case; the slope is almost zero, which implies that the effective-pressure coefficient is almost one. In contrast, the slopes of the bulk modulus for constant differential pressures are substantially larger compared to the slopes of the shear modulus, which means that the effective-pressure coefficient for the bulk modulus is significantly smaller than one.

Next, the method of Todd and Simmons is applied to the normalized shear and bulk moduli; as a result, an effective-pressure coefficient can be calculated using those data shown in Figure 6. While the effective-pressure coefficient for the bulk modulus is highly dependent on differential pressure (Figure 7), at P_d=14 MPa, the effective-pressure coefficient is 0.9, but drops to less than 0.3 at P_d=103 MPa; the effective-pressure coefficient for shear moduli remains near one (Figure 8), only dropping to 0.8 at 104 MPa differential pressure, which implies that shear-wave velocities are more sensitive to pore pressure changes than compressional velocities.

To better understand the implications of an effective-pressure coefficient, we combine equations (1) and (2), which leads to the following result:

$$P_e = P_d + (1-n) P_p \tag{6}$$

If we want to estimate changes in reservoir pore pressure (ΔP_p) from observed reservoir properties, we can now simply relate it to the change in effective pressure (ΔP_e):

$$\Delta P_e = -n \Delta P_p \tag{7}$$

Two conclusions can be drawn from equations 6 and 7. Because n is between zero and one, the effective pressure (P_e) is equal to or larger than the differential pressure (P_d). Second, any change in pore pressure (ΔP_p) is scaled with the effective-pressure coefficient. If the effective-pressure

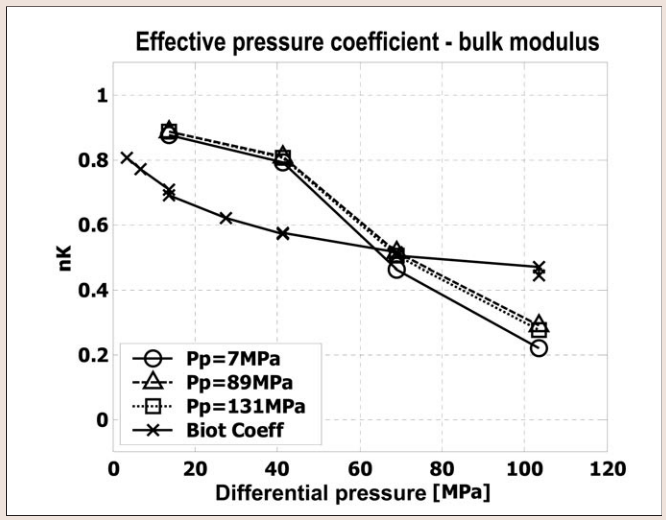

Figure 7. *Effective pressure coefficients for brine-saturated bulk moduli (nK) as function of differential pressure for different pore pressures in comparison to the Biot-Willis coefficient derived from the dry measurements. All brine-saturated data have been normalized to the properties of brine at P_p=3.9 MPa (1000 psi).*

coefficient is one, the change in pore pressure is equal to the change in effective pressure, while if n=0, a change in pore pressure will not affect the effective pressure and no change in rock properties can be observed.

The following example (Figure 9) illustrates the implications of the effective-pressure laws. The initial reservoir conditions are P_c=45 MPa and P_p=25 MPa. A pore pressure change of 10 MPa is induced during production, The differential pressure was 20 MPa and increases to 30 MPa, which results in a velocity increase from 3.65 km/s to 3.85 km/s. With 13% porosity, the effective-pressure coefficient for compressional waves is assumed to be 0.5. If we account for the effective stress law in this rock, the initial differential pressure is 32 MPa with an *effective pressure* change of 5 MPa. This results in a change in velocity of only 3.9 km/s to 3.95 km/s. Thus, the time-lapse signature would be a compressional wave velocity change of 200 m/s if n is assumed to be unity, but only 50 m/s when the effective-pressure coefficient is accounted for.

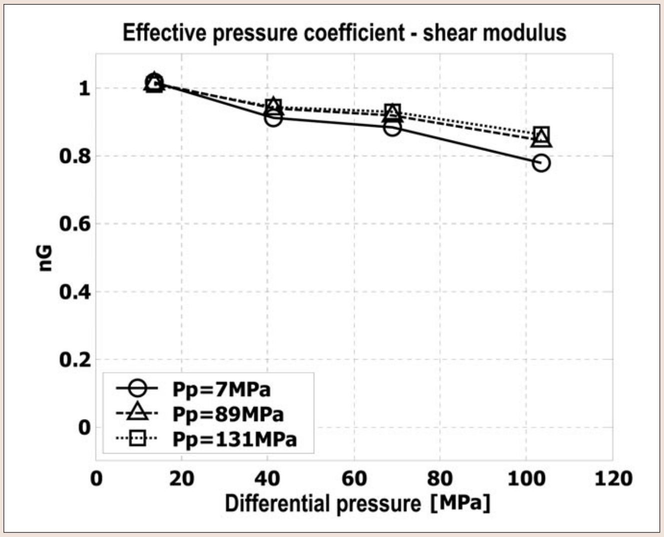

Figure 8. Effective pressure coefficients for brine-saturated shear moduli (nG) as function of differential pressure for different pore pressures. All brine-saturated data have been normalized to the properties of brine at P_p=3.9 MPa (1000 psi).

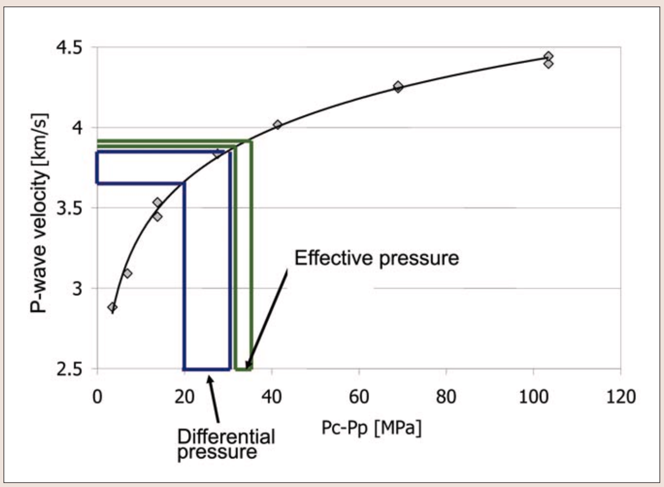

Figure 9. Compressional-wave velocity for a clastic sediment (ϕ=13%) as a function of stress, using differential and effective pressure laws. In this case, for n=1, a change in P_d of 10 MPa leads to a change of 3.65 km/s to 3.85 km/s in V_P (blue outline). In contrast, for an n=0.5, a decreasing pore pressure of 10 MPa, will result in an effective pressure change of 5 MPa and produce a much smaller change in V_P: from 3.9 km/s to 3.95 km/s (green outline).

Additional considerations. Beyond effective pressure, several other phenomena have to be considered to improve the understanding of the pressure dependencies of reservoir rocks. One of the largest uncertainties is how to upscale pressure effects. Very rarely is a statistically representative number of core samples measured. Also, the core selection can be very biased. Typically, our interest is focused on the best reservoir sands, and often we avoid sampling mixed lithologies, such as siltstones and shaly sands. But these mixed lithologies can be a major part of the reservoir and are affected by the change in reservoir pressure. Core measurements on lithologies like shaly sands are not very common, particularly because the time for pressure equilibration for these low permeability samples can be long. If the pressure effects for these lithologies are not known, it is wrong or at best difficult to upscale the pressure effects measured in the lab for the best reservoir rocks to reservoir and seismic scales. In addition, the pressure dependencies of shales are also not well understood. These sealing rocks will make up the majority of the stratigraphic column. On the timescale of reservoir production (years), pressure changes will diffuse into the surrounding shales. These tighter rocks need more investigation especially since changes in the overburden have now been documented (Hatchell, 2003).

Questions have been raised in recent years about how the coring process itself can influence the pressure dependencies of core samples. In the process of cutting and retrieving the core to the surface, the original state of stress is relieved and small cracks can be induced. Holt et al. (2000) show that on synthetic cores, the coring process and the resulting stress relief can have significant impact. Pressure cycling of the core samples before measurement will reduce the hysteresis, although the overall effects of sample damage caused during coring still remain.

Finally, more effort should be made to duplicate the true stress conditions within the reservoir and the overburden. Using only hydrostatic pressure gives an incomplete picture. Although measurements under hydrostatic conditions are easiest and most common, multi-axial loading should be applied.

For time-lapse studies of reservoirs with low porosities, the difference between differential and effective pressure has to be considered. Also, given the importance of permeability for reservoir performance, the relation between effective pressure and permeability changes needs to be investigated, in particular, in tight sandstone reservoirs.

Conclusions. Based on the measurements presented here, we conclude that the differential pressure oversimplifies the effective stress of elastic rock properties. The effective pressure (i.e., $P_c - nP_p$) better describes the stress dependence of rock properties, especially for rocks with low porosities.

While shear-wave velocities are approximately controlled by confining and pore pressure changes in the same way, changes in confining pressure affect more the compressional-wave velocities than the same changes in pore pressures. One implication of this result is that the inverted changes in reservoir pressure from time-lapse P-wave seismic data can significantly underestimate the real pressure changes in the reservoir.

Suggested reading. "Effect of pore pressure on the velocity of compressional waves in low-porosity rocks" by Todd and Simmons (*Journal of Geophysical Research*, 1972). "The influence of pore pressure and confining pressure on dynamic elastic properties of Berea Sandstone" by Christensen and Wang (GEOPHYSICS, 1985). "An experimental investigation of effective stress principles for sedimentary rocks" by Hornby (SEG 1996 *Expanded Abstracts*). "Seismic monitoring of pressure depletion evaluated for a UKCS gas reservoir" by MacBeth et al. (submitted to *Geophysical Prospecting*, 2005).

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