FREQUENCY DEPENDENT ELASTIC AND ANELASTIC PROPERTIES OF CLASTIC ROCKS

by
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ABSTRACT

This study focuses on measurements of clastic rocks ranging from 3 Hz up to 500 kHz in the laboratory and their application to well log analysis and a time-lapse study in the North Sea. Measurements of elastic properties over a large frequency spectrum can provide a better understanding of the dispersion and attenuation mechanisms in rocks and help to predict and model these effects.

Sandstones show a significant dispersion if inhomogeneities in saturation or open boundaries are present. These open boundaries cause a pore pressure equilibration and large changes in the compressibility of rocks. A large difference between the seismic and ultrasonic frequencies can be observed for the P-wave velocities if the rock is partially saturated with brine. The seismic frequency measurements follow the Gassmann prediction, while the ultrasonic measurement results track an arithmetic velocity average.

In the course of this work, I found a surprising result for shale. In the past, differences in velocities between VSPs and well logs, or well logs and ultrasonic laboratory measurements, have been reported, but no measurements have been done to cover the whole range from the seismic frequency to the ultrasonic frequency range. The results show that a shale with high porosity (26%), especially with the normal components of the stiffness tensor, show significant dispersion effects, while the shear stiffness shows no dispersion. Measurements of the normal and shear stiffness for a low porosity (8%) shale sample show no significant dispersion.

All these laboratory results have little value if not applied and confirmed in the “field”. The time lapse seismic experiment at Sleipner is a good example to observe
how the elastic properties change, if the rock is partially saturated. Carbon dioxide was sequestered in a shallow saline aquifer where the vertical migration of the $CO_2$ is controlled by shale barriers. Simple Gassmann predictions cannot explain the observed time-lapse changes. However, by including a pore pressure diffusion and its effect on the velocities, the observed anomalies can be better explained.
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Basic research is what I’m doing when I don’t know what I’m doing.

_Werner von Braun_
Chapter 1

INTRODUCTION

Several techniques have been applied in geosciences to understand properties of the subsurface. Seismic techniques have been one of the most successful methods used. The first successful deployment of exploration seismic was the Orchard field 1924 (Musgrave, 1967). Since then, seismic data has been used to explore and develop oilfields. The first application was to find salt domes in the subsurface. Later, seismic was used to identify and map the structure of the subsurface to find potential geological traps like anticlines. As seismic data quality improved, it was possible to use it to discriminate between brine-filled reservoirs and hydrocarbon-filled reservoirs (Hilterman, 1990). Derived Direct Hydrocarbon Indicators (DHI’s) are now common tools used to minimize risks in exploration for oil. Over the last 10 years, time-lapse seismic, also named 4D seismic, has been a useful tool in understanding reservoir complexities as well as fluid and pressure changes (Landro et al., 1999).

It is known from laboratory measurements and theoretical work that elastic and anelastic rock properties are correlated, and depend on how easy fluids can move within the porous rocks. Mobility of the fluids mainly depends on the permeability of the rock [Parra et al. (2002) and Pride et al. (2003)], the viscosity (Wang & Nur, 1988) of the fluid and fluid compressibility differences, if several fluids are present. White (1975), Dutta & Sheriff (1979), and Gist (1994) predicted what effects partial saturation would have on velocities and attenuation, which have been measured by Gregory (1976), Murphy (1984), and Cadoret et al. (1995). The quality of seismic
data is rapidly improving, which aids in the extraction of attenuation or frequency dependent elastic properties. These advancements allow us to test some of the proposed relations in the field.
1.1 Attenuation and Dispersion

Differences between well log derived velocities and velocities derived from seismic have been seen by Hornby et al. (1995), Hatchell et al. (1995), Schmitt (1999), and others. Hornby et al. (1995) observed a difference between velocities from well logs and velocities from seismic of 3% to 15% depending on lithology. One explanation for this observed difference is a scaling effect. This effect implies that different measurement techniques sample different volumes and materials. Another explanation is that the property is dispersive; this implies that the property is frequency dependent or wavelength dependent. For velocities, this means that a wave has a different propagation velocity depending on its wavelength.

Toksöz & Johnston (1981) give an excellent summary and introduction on the topic of attenuation and quality factor, which we will follow closely. Attenuation for a plane wave propagating through a homogeneous media can be described by an attenuation coefficient $\alpha$. This coefficient quantifies the exponential decay of amplitude:

$$A(x) = A_0 e^{-\alpha x}, \quad (1.1)$$

where $A$ is the amplitude at the position $x$ and $A_0$ is the amplitude at $x = 0$. This equation can be rewritten and solved for the coefficient $\alpha$, given two positions and the two corresponding amplitudes $A(x_1)$ and $A(x_2)$:

$$A(x) = \frac{1}{x_2 - x_1} \ln \left[ \frac{A(x_1)}{A(x_2)} \right]. \quad (1.2)$$

A common way to quantify attenuation is the quality factor $Q$, which is related to $\alpha$ by:

$$\frac{1}{Q} = \frac{\alpha v}{\pi f}. \quad (1.3)$$
The quality factor $Q$ can also be understood as the ratio between stored and dissipated energy:

$$Q = \frac{\omega \cdot E}{-dE/dt} = \frac{2\pi W}{\Delta W}, \quad (1.4)$$

with $E$ and $W$ as (instantaneous/elastic) energy and $dE/dt = \Delta W$ as the rate of energy loss.

A mathematical description of the stored and dissipated energy is the use of a complex modulus $M$, which is defined as the sum of the real part $M_R$ and imaginary part $M_I$:

$$M = M_R + iM_I. \quad (1.5)$$

Analogous to Equation 1.4, inverse $Q$ can be defined as the ratio of imaginary $M_I$ and real $M_R$ part of the complex modulus:

$$\frac{1}{Q} = \frac{M_I}{M_R}. \quad (1.6)$$

To understand the physical meaning of the quality factor $Q$, another relation is important:

$$\frac{1}{Q} = \frac{M_I}{M_R} = \tan (\Delta \delta) = \tan (\delta_\nu - \delta_\sigma), \quad (1.7)$$

which relates the phase difference $\Delta \delta$ between applied stress $\delta_\sigma$ and measured strain $\delta_\nu$ (cause and effect) to the quality factor $Q$.

For purely elastic material, no attenuation, the phase difference between stress and strain is zero ($\Delta \delta = 0^o$) and the quality factor is infinite. Oppositely, for a purely viscous material, a Newtonian fluid, the phase difference is: $\Delta \delta = 90^o$, which relates to a quality factor of zero. The more the material is “lossy”, the larger is the phase difference between stress and strain. A lot of Earth materials can be
described as viscoelastic materials with a finite and non-zero quality factor. O’Connell & Budiansky (1978) provide a very good overview on how the quality factor $Q$ is defined and how it can be measured for different measurement techniques.

Causality links attenuation and dispersion together, for a linear viscoelastic material, the Kramer-Kronig relation (Bourbie et al., 1987) relates modulus dispersion and the quality factor $Q$. No dispersion means no attenuation, conversely, no attenuation will result in constant properties as function of frequencies, (Dvorkin, 2005). The largest attenuation will occur at the frequency with the largest change in modulus. Several models exist to link the change in properties as function of frequencies with attenuation. Some of these relations (Mavko et al., 1998) are Maxwell’s model, a constant-$Q$ model, and Cole-Cole equation, to name a few.

Liu et al. (1976) introduced a nearly constant-$Q$ model, which correlates velocity, dispersion and attenuation:

$$\frac{V(\omega)}{V(\omega_0)} = 1 + \frac{1}{\pi Q} \ln \left( \frac{\omega}{\omega_0} \right).$$  

(1.8)

Here velocity at a frequency $V(\omega)$ is a function of the velocity at a reference frequency $V(\omega_0)$ and the quality factor $Q$. Figure1.1 shows the model with three different quality factors, which have been used to predict velocity changes as function of frequency. The lowest $Q$ relates to the highest attenuation, which shows the largest changes in velocity, from 2.8 km/s to 5.5 km/s. The opposite is also true: the highest $Q$ relates to the lowest attenuation and nearly constant velocity.

While the model by Liu et al. (1976) is based on an superposition of a lot of single attenuation mechanisms to create a nearly constant-$Q$ model, the Cole-Cole equation (Cole & Cole, 1941), describes the behavior of one single attenuation mechanism. Cole & Cole (1941) based their model on dielectric measurements on solids and liquids and
Figure 1.1. Near constant Q-model (Liu et al., 1976): modeled velocity change as function of frequency for three different Q’s (10, 50, 250)
it has been applied to elastic properties, e.g. Casula & Carcione (1992) and Toverud & Ursin (2005). The Cole-Cole equation couples dispersion and attenuation as indicated by a real \((M_R)\) and imaginary \((M_I)\) component of a modulus:

\[
M(\omega) = M_\infty + \frac{M_0 - M_\infty}{1 + (i\omega\tau)^{1-\kappa}} = M_R(\omega) + iM_I(\omega),
\]

with \(\tau\) being the relaxation time and \(\kappa\) being a distribution parameter that gives the range over which a relaxation mechanism operates. \(M_\infty\) and \(M_0\) are the infinite and zero frequency moduli, respectively. Figure 1.2 shows an example of the Cole-Cole model with \(M_\infty = 15\) GPa and \(M_0 = 10\) GPa, and \(\tau = 10^{-3}\) s. The distribution parameter has been varied between 0.2 and 0.8, which results in that the low and high frequency limits are independent of the the distribution parameter, but that the rate of change of the modulus are largely dependent of this parameter. A distribution parameter of 0.8 results in a very slow change in the modulus over a large frequency range, while a distribution parameter of 0.2 results in a change in modulus of a narrow frequency range. Since the amount of attenuation, which is measured as inverse \(Q\) in Figure 1.2, is directly related to how fast the modulus is changing, it is no surprise that the largest attenuation is for the distribution parameter of 0.2. Chapter 4 will describe in more detail the different input parameter for the Cole-Cole equation.
Figure 1.2. Influence of varying the distribution parameter ($\kappa$) on the dispersion and loss.
1.2 Dispersion and Attenuation Mechanisms

To understand dispersion mechanisms, elastic properties have to be measured over a large range of frequencies. Figure 1.3 shows a comparison between several measurement techniques, frequency range, and applied strains. Different disciplines use measured elastic properties to resolve specific problems in their field: rock mechanics to predict borehole failure, petroleum engineering to understand reservoir compaction, and rock physics to understand time-lapse seismic observations. Seismic frequency measurements are conducted at low frequencies and at low strains, while measurements in rock mechanic laboratories are at the same or lower frequencies, but at much larger strain. On the other hand, ultrasonic measurements are at the same strain level as seismic measurements, but are conducted at much higher frequencies.

Iwasaki et al. (1978) shows one example of the dependency of elastic properties on the strain amplitude. In this specific case, he investigated the shear stiffness of unconsolidated sands. Figure 1.4 shows that the increase of strain from $10^{-4}$ to $10^{-3}$ relates to a decrease of the shear modulus of 50%, oppositely, the shear modulus increases by nearly 50% with decreasing strain amplitude from $10^{-4}$ to $10^{-6}$. The shear modulus is nearly independent of the strain amplitude if the strain is smaller than $10^{-5.5}$.

To compare results of different measurement techniques it is important to measure at the same frequencies, but with different strain amplitudes, to investigate non-linear elastic behavior of materials. To investigate the dispersive behavior of materials, the measurements should be done at different frequencies, but with the same strain amplitudes. Changing both strain amplitude and frequency in one experiment will make it very difficult to distinguish between non-linear deformation and dispersion, since non-linear deformation can be seen as apparent deformation.
Figure 1.3. Schematic where different measurement techniques are situated in frequency-strain space
Figure 1.4. Shear modulus versus strain for unconsolidated sand normalized to the strain amplitude of $10^{-4}$, from Iwasaki et al. (1978)
In the case of seismic, the applied strain to the rock should be lower than $10^{-6}$ to deform the rock elastically. Low frequency measurements done at the Colorado School of Mines (Batzle et al., 2006) are in a frequency range between 3 Hz and 2000 Hz, with measured strains lower than $1 \cdot 10^{-7}$ to avoid non-linear deformations. As a comparison, if you compress Mount Everest (8872 m) vertically, a strain of $1 \cdot 10^{-7}$ would correspond to a deformation of 0.9 mm over its height. These measurements at very small strains and over a large frequency range (3-2000 Hz) and at ultrasonic frequencies (500 kHz) allows us to investigate dispersion and attenuation mechanisms.

Figure 1.5 shows the current understanding of dispersion mechanisms of a porous rock filled with fluids. The frequency behavior of fluid filled rocks depends on the mobility of the fluids within the porous frame of the rock. At very low frequencies and open boundary conditions, the pore pressure within the rock can equilibrate and the pore fluid is not supporting the rock frame, as a result, the drained elastic properties are equal to the properties of a room dry rock. Gassmann (1951) described this behavior in one chapter of his paper.

At higher frequencies, the pore pressure can equilibrate partially. Pride & Berryman (2003), Pride et al. (2004), and Shapiro & Müller (1999) describe frequency dependent velocities and attenuation caused by pressure equilibration induced by inhomogeneities of the rock or the pore fluid distribution. For closed boundary conditions, Gassmann (1951) described the behavior of fluid filled porous rock: where the the pore fluid is stiffening the rock, the bulk modulus increases.

Figure 1.6 shows bulk modulus measurements of a porous sandstone both dry and saturated with brine as a function of frequency, under open and closed boundary conditions. The bulk modulus is the lowest for the dry rock and increases substantially if the rock is saturated with brine. For the closed boundary condition, the rock
Figure 1.5. Relative position of different relaxation mechanisms versus frequency, or fluid mobility.
Figure 1.6. Relative position of different relaxation mechanisms versus frequency, or fluid mobility.
is strengthened. If the boundary is open, the bulk modulus is decreasing at low frequencies to reach the value of the drained/dry bulk modulus.

However, a substantial change of velocities can be obtained by increasing the frequency, where the fluid pressure inside the rock is not in equilibrium. This is an unrelaxed state to where different dispersion mechanisms, like local flow by Mavko & Nur (1975), and O’Connell & Budiansky (1977) or the Biot mechanism (Biot, 1956), may have a large influence. For a porous sandstone like Berea sandstone, the Biot attenuation would occur at ultrasonic frequencies, around 1MHz, while local flow mechanisms would dominate around 10-100 kHz. For open boundary conditions, the shift from macro flow to relaxed conditions (where Gassmann’s equation is valid) is within the seismic frequency range.

At very high frequencies, another attenuation mechanism has been observed: viscoelastic loss within the pore fluid. Figure 1.7 shows the Young’s modulus quality factor measured for a carbonate rock filled with heavy oil (API=-5) and for the heavy oil itself. The quality factor is decreasing with increasing frequency, with the quality factor of the oil itself five times larger compared to the quality factor measured for the rock with the oil inside. Multiplying the measured $Q$ of the oil with the porosity of the rock fits the measured loss in the rock very well, which indicates that the measured $Q$ of the rock is caused not by rock-fluid interaction but by the loss within the the heavy oil itself. The differences between the measured $Q$ of the rock and the scaled $Q$ of the oil can be caused by the extraction process of the oil from the rock. These viscoelastic attenuation mechanisms within the pore fluid itself are not accounted for in geophysics currently, however, these mechanisms seem to be important in characterizing heavy oil deposits in Canada.
Figure 1.7. Young’s modulus quality fractar measured for a carbonate rock saturated with heavy oil (Uvalde carbonate) and for the heavy oil itself at 25°C.
1.3 Thesis Outline

Elastic and anelastic properties as function of frequency of fluid saturated rocks are the focus of this thesis, which has two major part, each of them consists of two independent papers, which co-authored. The first section focuses on experimental studies to measure frequency dependent elastic properties and the second section concentrates on the application of this knowledge in field studies to better predict and understand observed seismic data.

Measurements between 3 and 2000 Hz and at ultrasonic frequencies (500 kHz) have been conducted at the Colorado School of Mines, using a low frequency device which is similar to device by Spencer (1981). These measurements are not very common [Spencer (1981), Paffenholz & Burkhardt (1989), Mörig et al. (1996), and Suarez-Rivera et al. (2001)] and very time consuming compared to ultrasonic measurements, but these low frequency measurements have the advantage that they cover a broad frequency range including seismic frequencies. The first paper investigates dispersion in sandstones for different fluid types and saturations. Even for a very soft liquid like CO₂, differences between high and low frequencies can be observed. All five independent components of the stiffness tensor of two shales are the topic of the second experimental paper. Large differences for the normal stiffnesses as function of frequency have been observed for the shale with a high porosity, while a much smaller dispersion is observed for the shale with low porosity. Several other measurement techniques, like seismic and micro-indentation, confirm these observations.

The second section of the thesis is focused on the application of the newly gained understanding of frequency dependent elastic properties. Synthetic seismograms derived from well logs are often used to calibrate seismic data. One of the remaining challenges in creating synthetic seismograms is how to down-scale the velocities de-
rived from well logs at 10 kHz to seismic frequencies at 60 Hz. In the third paper, we show a methodology to predict the velocities at seismic frequencies. Finally, the last paper focuses on the time-lapse study of the $CO_2$ injection project at Sleipner, as gas field in the North Sea. An improved rock physics model, including pressure diffusion at the interface between the brine and $CO_2$, was developed to predict the time shifts below injected saline aquifer. We propose in this paper a frequency dependent time-shift caused by pressure diffusion at the brine and $CO_2$ interface to better explain the mismatch between observed and calculated mass of $CO_2$ in the saline aquifer.

The work in this thesis shows that not only the rock and fluid properties affect the observed dispersion and attenuation, but also the boundary conditions and inhomogeneities of this rock-fluid system are important model, invert, and understand elastic properties.
Chapter 2

PORE FLUID MOTION AND FREQUENCY-DEPENDENT ELASTIC PROPERTIES: EXPERIMENTAL EVIDENCE

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Velocity dispersion from seismic to ultrasonic frequencies is typical of porous reservoir rocks. Dynamic elastic properties between 3 Hz and 500 kHz were measured for high porosity and permeability sediments. The magnitude and frequency range of dispersion is dependent on the permeability of the rock and on the viscosity and compressibility of the fluid. Minimal dispersion was found for vacuum dried samples. The maximum measured dispersion was between 10 Hz and 500 kHz for partially brine-saturated samples. These effects of inhomogeneous saturation distributions are evident and produce a substantially higher ultrasonic velocity at partial-brine saturations compared to measurements at seismic frequencies.
2.1 Introduction

From seismic data, we can derive two independent elastic properties and density. But this information may not be enough to resolve for the fluid content and saturation. Fluid effects on seismic velocities are commonly observed and discussed in literature. Wood’s equation (1955), Gassmann’s equation (1951), and Brown & Korringa (1975), describe a frequency independent behavior (low frequency limit). In contrast, several theories are available to describe frequency-dependent elastic properties, including Biot’s (1956) inertial model, the crack model of O’Connell & Budiansky (1977), and the squirt model of Mavko & Nur (1975), and Mavko & Nur (1979). Fluid viscosity and rock permeability have an important influence on the velocity dispersion of the rock/fluid system, and on the frequency range at which the maximum dispersion, or critical frequency, occurs. Sonic logs (2-20 kHz) typically measure higher velocities compared to seismic (10-100 Hz).

The focus of this paper is the laboratory measurements ranging from 3-3000 Hz, in comparison to measurements at ultrasonic frequencies 500 kHz and above. Flow properties of the rock (permeability), properties of the fluid (compressibility and viscosity) and boundary conditions influence the elastic properties. Overall, the dispersion, if recognized, can help to identify both the fluid and hydraulic properties of the rock.

In this chapter, first we will summarize pressure dependency of density and incompressibility (modulus) of the fluids used in the experiments (carbon dioxide, butane, and brine) and discuss what influence the mobility has on different dispersion mechanisms. Next, the experimental setup for the low-frequency measurements is shown in detail to distinguish the differences of this technique to typical wave propagation measurements. Finally, the results are shown and discussed.
2.2 Fluid and Flow Properties

2.2.1 Bulk Modulus

Properties of pore fluids have a strong influence on the bulk rock properties for compliant or high porosity rocks. Gas compressibility is high and the bulk modulus is low. The modulus increases rapidly as it crosses the phase boundary from gas to liquid. The magnitude of this change is fluid dependent. Figures 2.1 and 2.2 show density and bulk modulus as function of pressure for three different fluids: brine with a salinity of 50,000 ppm, butane, and carbon dioxide ($CO_2$).

These fluids were selected to provide measurements over a large range of fluid bulk moduli. The program SUPERTRAPP 2.0 (SUPERTRAPP, 1998) was used to calculate properties of pure butane and carbon dioxide. Brine properties were calculated using FLAG 3.1 (Han & Batzle, 2000), a program based on improved equations first established by Batzle & Wang (1992).

Brine does not go through a phase change in the plotted pressure range and has a nearly linear behavior (Figures 2.1 and 2.2). In contrast, a large change in properties occurs while crossing the gas-liquid phase boundary for butane and $CO_2$. $CO_2$ shows an interesting behavior: The ratio between the bulk moduli of brine and $CO_2$ (g) at 5 MPa is nearly 400, and the ratio drops to 8 at 15 MPa, where $CO_2$ is in the liquid phase. At the same pressure, the ratio of densities between brine and $CO_2$ drops from 7 (at 5 MPa) to 1.2 (at 15 MPa). In comparison to brine, the $CO_2$ is much more compressible in the liquid phase, while its density is nearly equal to the density of brine. This observation is important for fluid substitution models, where the bulk modulus of the fluid is one of the basic input parameters.
Figure 2.1. Brine, butane and carbon dioxide density as a function of pressure ($T = 20^\circ C$)
Figure 2.2. Brine, butane, and carbon dioxide modulus as a function of pressure (T = 20°C)
2.2.2 Fluid Mobility Effects

As a seismic wave passes through a porous material, the slight compression of the rock causes fluid motion. A maximum in the fluid motion occurs when the ability of the fluid to move is “tuned” to the seismic frequency. This behavior is similar to a damped resonance, where the frequency depends on the fluid mobility. Fluid mobility depends not only on the fluid properties (primarily viscosity, $\eta$), but also on the transport properties of the rock, the permeability ($k$), which is controlled by the geometry (interconnectivity and tortuosity) of the flow channels in the rock. The fluid mobility, $m$, is defined as:

$$m = \frac{k}{\eta}.$$  \hspace{1cm} (2.1)

Note, sometimes in hydrogeology the hydraulic conductivity is named mobility, which is differently defined, including the density and acceleration of gravity.

In general, for a single relaxation mechanism to describe frequency dependent elastic properties and attenuation, there is a cross-over frequency, $\omega_c$, separating the low-frequency, “relaxed” regime from the high-frequency “unrelaxed” regime. Biot (1956) and the model of O’Connell & Budiansky (1977) are two models that predict these effects. However, they predict the opposite dependencies of this cross-over frequency as function of viscosity. O’Connell & Budiansky (1977) predict that an increase in viscosity will shift the cross-over to lower frequencies:

$$\omega_c \approx \frac{K}{\eta} \left( \frac{c}{a} \right)^3 \approx \frac{K}{\eta} \alpha^3,$$  \hspace{1cm} (2.2)

here, $K$ is the rock bulk modulus, $\omega_c$ the cross-over, and $\alpha$ the pore aspect ratio defined as the thickness to diameter ratio of an oblate spheroidal crack. In contrast, Biot (1956) predicts a shift to higher frequencies for increasing viscosities. Biot’s
definition of the cross-over frequency $\omega_c$ contains porosity, $\rho_{fl}$ fluid density, and $k$ permeability as independent variables:

$$\omega_c \approx \frac{\eta \phi}{k \rho_{fl}}$$

(2.3)

Note that the viscosity term is in the numerator of the expression in Biot’s model (1956) and in the denominator in O’Connell and Budiansky’s model (1977).

Equation 2.3 indicates that fluid mobility not only depends on the fluid viscosity but also depends on permeability, which controls how fast pore pressure pulse can equilibrate within a rock. At some permeability, the cross-over occurs between seismic and sonic logging frequencies, potentially resulting in a mismatch between these two measurements. It is important to note that permeability can vary, depending on the pore fluid chemistry and the clay content of the rock. Batzle et al. (1999b) and Batzle et al. (2006) have shown examples of permeability changes over two orders of magnitude caused by changes in the salinity of the injected brine. At high salinity, the clay minerals were collapsed, which increased the connectivity between the pore space and resulted in a higher permeability. At low salinities, the clays expand, plugging the pore space and making fluids immobile. These kinds of rock property alterations are well known, but the effect on the elastic properties needs further investigation.

The permeability effects can be predicted by Biot’s high frequency model, Equation 2.3. In this theory, with increasing permeability, the cross-over frequency will shift to lower frequencies. If the permeability is very low, as in some gas reservoirs, only a small dispersion effect should be observed between seismic and acoustic logging velocities. This is not the case for some high permeability reservoirs. In that situation, the peak attenuation can occur between seismic and logging frequency range, which can explain differences between velocities derived by seismic and logging (Bat-
In addition, surface tension and wettability can affect the elastic rock properties, Clark et al. (1980), Tittmann et al. (1980), and Tserkovnyak & Johnson (2002); however these effects are poorly understood. In the last ten years, a great amount of effort has been focused on modeling inhomogeneities in porous materials and their effect of these heterogeneities on frequency dependent properties, like velocity and attenuation. Several factors can cause inhomogeneities within a porous rock. For example, inhomogeneities can be caused by partial saturation. Carcione et al. (2004) and Pride et al. (2004) describe frequency dependent velocities and attenuation or inhomogeneities caused by geological processes. Gurevich et al. (1997) and Pride et al. (2002) describe the effect of layered porous rocks while heterogeneities within the pore structure are described by Shapiro & Müller (1999) and Pride & Berryman (2003). All these dispersion mechanisms are caused by pore fluid flow and diffusion of pore pressure. Later we will see that this pore pressure diffusion process has a significant effect on our acquired low-frequency data.
2.3 Measurement Method

2.3.1 Low Frequency Measurements

Experiments of wave propagation in porous rocks are not feasible for the exploration seismic frequency range (10-100 Hz) in the laboratory because of the large wave lengths. Therefore such “low frequency” measurements are based on the linear stress-strain relationship: Hooke’s law [(Spencer, 1981), (Paffenholz & Burkhardt, 1989), and (Mörg et al., 1996)]. The design of the measurement apparatus is shown in Figure 2.3 and is similar to the apparatus used by Spencer (1981).

A sinusoidal (or cosinusoidal) stress is applied by a solenoid-driven shaker to a column composed of a rock sample and end caps made from a standard material, which is, in our case, aluminum. A detailed setup of the rock sample with the end caps is shown in Figure 2.4. Since nitrogen gas is used as the confining fluid, a thin layer of epoxy and polyimide film (Kapton®) are attached to the sample to prevent gas diffusion into the sample and to permit 100% pore fluid saturation inside the sample. These layers of epoxy and Kapton® are very thin in comparison to the sample diameter; their effect on the strain measurement can be neglected, which was confirmed by calibration tests measuring aluminum standards with and without a Kapton® layer. Micro-valves on the top and the bottom of the samples allow fluid substitutions, while the sample stays under confining pressure and closes the boundaries of the sample.

An applied sinusoidal stress field will result in sinusoidal deformation and strain, which is measured by strain gages mounted vertically on the aluminum standard and vertically and horizontally on the rock sample. Using horizontal and vertical gages on the rock sample allows us to calculate Young’s modulus and Poisson’s ratio. Young’s modulus of the rock, $E_{rx}$, is calculated from the ratio of the strain amplitudes between
Figure 2.3. Schematic of the low-frequency measurement apparatus
Figure 2.4. Detailed sample setup showing the rock sample and end caps
the vertical gages on the rock, $\epsilon_{rx}^3$, and the aluminum standard, $\epsilon_{al}^3$:

\[
E_{rx}\epsilon_{rx}^3 = \sigma_{rx/al}^3 = E_{al}\epsilon_{al}^3
\]

\[
E_{rx} = E_{al}\frac{\epsilon_{al}^3}{\epsilon_{rx}^3},
\]

(2.4)

where $\sigma_{rx/al}^3$ is the stress applied to the rock / aluminum column. Poisson’s ratio, $\nu_{rx}$, is the ratio between horizontal, $\epsilon_{rx}^1$, and vertical strain, $\epsilon_{rx}^3$, measured on the rock under uniaxial $\sigma^3$ loading:

\[
\nu_{rx} = -\frac{\epsilon_{rx}^1}{\epsilon_{rx}^3}.
\]

(2.5)

Under the assumption of homogeneity and isotropy, any two measured independent elastic properties are enough to describe the elasticity of the rock. As a result, bulk and shear moduli ($K$, $G$) are derived from the measured Young’s modulus and Poisson’s ratio:

\[
K = \frac{E}{3(1-2\nu)},
\]

\[
G = \frac{E}{2(1+\nu)}.
\]

(2.6)

Finally, compressional and shear wave velocities are then calculated using the known rock density, $\rho$:

\[
v_P = \sqrt{\frac{K + 4/3G}{\rho}},
\]

\[
v_S = \sqrt{\frac{G}{\rho}}.
\]

(2.7)

Simultaneously with the low-frequency measurements, ultrasonic compressional and shear wave velocities are collected. These are pulse transmission measurements at
a dominant frequency around 500 kHz. A piezoelectric transducer produces a mechanical impulse or deformation. This elastic pulse is transmitted along the axis of the rock. A receiving transducer transforms this deformation back into the recorded electrical pulse (Figure 2.4). Velocities are derived from the transit time between the transmitted and received signals and the known sample length.

### 2.3.2 Error Analysis

Differences in measured elastic values between the low-frequency measurements and the ultrasonic measurements can be partly due to the different scales of these measurement types. Low-frequency measurements using strain gages can be considered point measurements: the strain gages cover only a very small area on the sample surface. In contrast, the pulse transmission technique measures averaged bulk elastic properties of the sample. This means that inhomogeneities in the sample and gage placement can cause differences between the low and high frequencies even for dry measurements. If the dry measurements do not show dispersion between the low-frequency measurements and ultrasonic frequency measurements, the measured dispersion for fluid filled rocks must be caused by the presences of fluids and not sample heterogeneity.

Error estimations for low-frequency measurements are difficult (see Gautam (2003a)). Two different types of error can be distinguished: random and systematic. Random errors are caused by the gage placements, including rotation of the gages, misalignment of the sample column, and noise due to the cross talk of wires. Systematic errors are caused the amplifiers (we use 8 different channels), resonances in the experimental setup, and the error in the resistance of the strain gages. Repeatability tests showed that these strain measurements are repeatable within 1%.
To estimate the error of these low-frequency measurements we calculated the difference between the two pairs of gages with the same orientation. In the ideal case, the difference between the two vertical (horizontal and aluminum) strain gages are zero. Every difference is measure of the error in the measurements. Figure 2.5 shows the strain gage error in percent for all three kinds of gages, the gages on the aluminum, vertical and horizontal gages on the rock. The smallest error are on the vertical gages, the error on the aluminum gages is 5 times larger in comparison, the error of the horizontal gages are ten times larger.

Using the measured errors for these three kinds of gages using Equations 2.4 to 2.6, the error in Young’s modulus, Poisson’s ratio, shear and bulk modulus can be estimated, shown in Figure 2.6. The smallest percentage error is error for Young’s modulus, followed by the error for the shear modulus and the Poisson's ratio, while the error for the bulk modulus is the largest. Additional, the error for the bulk modulus is dependent on the Poisson’s ratio, for low ratios (0.2), the maximum error is 10%, while for high ratios (0.4), the error can reach more then 30% for the same gage errors.

Gautam (2003a) concluded also that the smallest errors for the low-frequency measurements are for Young’s modulus and shear modulus. Due to error in Poisson’s ratio, the errors for bulk moduli are the largest. Errors in the ultrasonic measurements can be due to the error in picking the travel time, inaccurate length measurements, and by changes in the pulse shape. An error analysis of the ultrasonic measurements shows that the measured ultrasonic velocities have an error smaller than 1% (Ecke, 2004).
Figure 2.5. Errors (in percent) in strain amplitude measurements for the aluminum gages, vertical gages and horizontal gages.
Figure 2.6. Estimated errors (in percent) for Young's modulus, Poisson’s ratio, shear and bulk modulus as function of frequency. Top shows the error for a rock with a Poisson’s ratio of 0.2, bottom shows the error for a rock with a Poisson’s ratio of 0.4.
2.4 Low Frequency Measurement

2.4.1 Sample Characterization

The samples used in this study are clastic sediments from the North Sea. Table 2.1 gives an overview of rock properties. Four different cores were measured with very high porosity and permeability. Sample X #2 and Y #1 are soft in comparison to the better cemented and more consolidated samples Z #5 and #13. Effective porosity, permeability, and grain density were measured by a commercial core laboratory. Total porosity was measured from the bulk and grain volume, using Archimedes’ principle and Boyle’s law. Steady-state air permeability measurements were made at 400 psi (2.8 MPa) confining pressure. Changes in the differential pressure will change porosity and permeability. For sample X #2, the porosity reduction as a function of differential pressure showed a porosity decreased from 29.1% at 1 MPa to 27.6% at 35 MPa.

Analysis of mineral composition of the samples showed quartz, K-feldspar, and plagioclase, as well as minor amounts of muscovite and pyrite. The material varied widely in compaction and cementation. For example, the major contrast between samples X #2 and Y #1 are differences in clay content acting as cement. Sample X #2 has 6% kaolinite compared to sample Y #1, which has less than 1%. This difference is noticeable in the texture of these two samples. Sample X #2 is shown in the Scanning Electron Microscope (SEM) images (Figure 2.7), where poor consolidation is obvious. Micro fractures have been observed, but it is very difficult to distinguish induced fractures (caused by drilling, cutting, and polishing processes) from naturally occurring fractures. Some fractures show traces of remineralization on the walls, indicating that at least some of the fractures are natural. Figure 2.8 show SEM pictures of samples Z #5 and #13, respectively. While the SEM picture of sample
Figure 2.7. SEM pictures of the pore structure of Sample X #2 (left: amplification of 50x right: amplification of 200x) 1 - quartz grain, 2 - pore space, 3 - clay, 4 - grain contacts

X #2 shows mostly loose, rounded grains, samples Z #5 and #13 show a more ridged matrix caused by quartz cementation, which supports the grain structure. The pore structure is also a good permeability indicator with the pore connectivity of sample X #2 being much higher than the pore connectivity of samples Z #5 and Z #13. Measurements of permeability listed in Table 2.1 confirm this observation. The two samples exhibit two orders of magnitude difference in permeability, 3.55*10^{-12} m^2 (3600mD) for X #2 versus 2.57*10^{-14} m^2 (26mD) for Z #5. Note that the SEM pictures of samples X #2, Z #5 and Z #13 in Figures 2.7 (left) and 2.8 are at the same scale.

2.4.2 Results: Sample X #2

Measurements were conducted on this sample under conditions of vacuum-dry, CO_2, and butane-saturation at a differential pressure of 6.9 MPa and at room temperature. Figure 2.9 shows the measured compressional and shear wave velocities for
Figure 2.8. SEM pictures of the pore structure of Sample Z #5 (left) Sample Z #13 (right) (amplification of 50x)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Porosity [%]</th>
<th>Permeability [m²]</th>
<th>Permeability [mD]</th>
<th>Grain Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X #2</td>
<td>29.1</td>
<td>3.55 * 10⁻¹²</td>
<td>3600</td>
<td>2607</td>
</tr>
<tr>
<td>Y #1</td>
<td>34.9</td>
<td>8.64 * 10⁻¹²</td>
<td>8750</td>
<td>2363</td>
</tr>
<tr>
<td>Z #5</td>
<td>17.1</td>
<td>2.63 * 10⁻¹⁴</td>
<td>26.6</td>
<td>2648</td>
</tr>
<tr>
<td>Z #13</td>
<td>14.0</td>
<td>9.18 * 10⁻¹⁶</td>
<td>0.93</td>
<td>2648</td>
</tr>
</tbody>
</table>

Table 2.1. Petrophysical parameters of the investigated rocks
the rock sample measured first under dry conditions (diamonds), then saturated with $CO_2$ in the liquid phase (squares) at a pore pressure of 13.8 MPa. Following that, the pore pressure was decreased under the condition of constant differential pressure, until the $CO_2$ entered the gas phase (3.5 MPa) (triangles). The rock saturated with $CO_2$ in gas-phase shows nearly the same behavior as dry. Decreases in P- and S-wave velocities observed for the measurements with the liquid $CO_2$ are caused by the high density of the liquid-phase $CO_2$ and the very low bulk modulus compared with Figures 2.1 and 2.2. Here, density dominates the fluid effect for these velocity measurements. Resonance of the mechanical parts of the measurement system affect the data, especially at high frequencies as seen at 1000 Hz (Figures 2.9 and 2.10).

Next, the sample was vacuumed and liquid butane was injected at 6.9 MPa fluid pressure. Two sets of measurements with different pore pressure conditions were acquired: butane in a liquid phase at 6.9 MPa (squares) and in a gas phase at 0.1 MPa (triangles) shown in Figure Figures 2.10. Both the compressional and shear wave velocities decrease in comparison to the measured dry velocities. The decrease of the shear wave velocities is caused by the increase of density of the pore fluid going from gas to liquid, with the lowest velocity for the rock filled with liquid butane. However, this is not the case for the compressional wave velocities, where the velocities for the gas filled rock and liquid butane filled rock are nearly the same. This difference between the shear and compressional wave velocities can be explained by the substantial change of the butane bulk modulus $K_{fl}$ increasing from 0.1 MPa (gas) to 550 MPa (liquid), results in a stiffening of the rock frame and increase of the compressional velocity. So far, all measurements we described have been done with closed pore fluid valves, which means closed boundary conditions.

Both the top and the bottom of the sample have valves to allow fluid substitution
Figure 2.9. P- and S-wave velocity for Sample X #2 saturated with CO$_2$ under constant differential pressure (Pd=6.9 MPa) and for different pore pressures (dry - diamonds, 3.5 MPa (gas) - triangles, 13.8 MPa (liquid, valve closed) - squares)
Figure 2.10. P- and S-wave velocity for Sample X #2 saturated with butane under constant differential pressure (Pd=6.9 MPa) and for different pore pressures (dry - diamonds, 0.1 MPa (gas) - triangles, 6.9 MPa (liquid, valve closed) - squares)
while the rock sample remains under confining pressure. With the micro valves open (see Figures 2.3), fluid can equilibrate with the outside pore fluid system. As the sample is compressed, pore pressure increases slightly and stiffens the rock (Gassmann effect); however, if there is sufficient time, pore fluid movement through the pore fluid valves will prevent pore pressure increase and the subsequent stiffening of the rock frame. If the frequency is low enough, the pore fluid can equilibrate and the stiffness of the rock is similar to the drained stiffness. Note that this behavior requires macroscopic fluid pressure equilibration. Conversely, if the applied frequency is high, the pore fluid cannot equilibrate with the outside, which creates a pore pressure increase inside the rock sample; as a result, the fluid contributes to increase the total stiffness of the rock fluid system.

Earlier, the concept of pore pressure equilibration was introduced; elastic properties are not just dependent on the fluid modulus but also on the boundary conditions, i.e., if the fluid is in drained or undrained conditions. As a result of this change from drained conditions to undrained conditions, the data show step-like frequency dependence at about 200 Hz with the open valve for liquid CO$_2$ and liquid butane, respectively. The effect is more noticeable when we plot Young’s modulus for these measurements in Figures 2.11 and 2.12. This effect is even more noticeable looking at the Poisson’s ratio instead of Young’s modulus. Figures 2.13 and 2.14 (circles) explicitly show this behavior for CO$_2$ and butane.

Below 10 Hz the CO$_2$ is in the drained state; above this frequency, the undrained state dominates. Most importantly, the data at higher frequencies match the data acquired with the closed valve (squares), where macroscopic fluid pressure equilibration is prevented and the undrained state dominates. The CO$_2$ data in Figure 2.13 show this transition from drained to undrained at lower frequencies and with a lower mag-
Figure 2.11. Young’s Modulus for Sample X #2 dry and saturated with CO₂ under constant differential pressure (Pd=6.9 MPa) and for different pore pressures (dry - diamonds, 3.5 MPa (gas) - triangles, 13.8 MPa (liquid, valve closed) - squares, 13.8 MPa (liquid, valve opened) - circles)
Figure 2.12. Young’s Modulus for Sample X #2 dry and saturated with butane under constant differential pressure (Pd=6.9 MPa) and for different pore pressures (dry - diamonds, 0.1 MPa (gas) - triangles, 6.9 MPa (liquid, valve closed) - squares, 6.9 MPa valve (liquid, valve opened) - circles)
Figure 2.13. Poisson’s Ratio for Sample X #2 saturated with \( CO_2 \) under constant differential pressure (\( P_d = 6.9 \) MPa) and for different pore pressures (dry - diamonds, 3.5 MPa (gas) - triangles, 13.8 MPa (liquid, valve closed) - squares, 13.8 MPa (liquid, valve opened) - circles)
Figure 2.14. Poisson’s Ratio for Sample X #2 saturated with butane under constant differential pressure (P_d=6.9 MPa) and for different pore pressures (dry - diamonds, 0.1 MPa (gas) - triangles, 6.9 MPa (liquid, valve closed) - squares, 6.9 MPa valve (liquid, valve opened) - circles)
magnitude of change, compared to butane (Figure 2.14). This difference can be explained by the difference in fluid modulus (Figure 2.2) and viscosity. Thus, the frequency at which the step occurs is a measure of or is related to the fluid, while the magnitude of the step depends on the bulk modulus of the fluid and the rock, as well as on the porosity of the rock. Although Young’s modulus shows some fluid effects, Poisson’s ratio seems to be a better indicator, as shown in Figures 2.13 and 2.14.

2.4.3 Results: Sample Y #1

Variations of velocity with saturation were also measured on this high porosity sample. Under dry conditions we get expected low velocities for P- and S-waves. The velocities are in the range of 2.24 km/s at 3.4 MPa to 2.91 km/s at 20.7 MPa for compressional velocities and 1.36 km/s to 1.71 km/s for shear wave velocities for a range of frequencies from 5 Hz to 500 kHz. The differential pressure was then kept constant, and brine with a salinity of 75,000 ppm was injected in steps. This injection sequence was used to simulate partial saturation by fluids at reservoir conditions. The results are shown in Figures 2.15 and 2.16.

Curves are plotted with triangles representing the measured data points for the low frequency range. The low frequency data points are the averaged measurements in the frequency range from 5 to 50 Hz. Squares indicate ultrasonic frequency measurements at 500 kHz. The behavior of the velocities for both frequency ranges is very different. While the low frequency measurement follows the Gassmann prediction (heavy green dotted line), the ultrasonic measurements are bounded by a travel time average more like Wood’s (1955) equation (light dashed line). The difference between the measured low frequency data and the prediction made by Gassmann’s equation can be explained by errors in the estimated mineral bulk modulus and in
Figure 2.15. P-wave velocity in a North Sea sandstone at a differential pressure of 6.9 MPa as a function of brine saturation. Data are shown for low frequencies (triangles) and ultrasonic frequencies (squares). The dashed line shows a velocity average (Wood’s equation) and the dotted line shows a Gassmann substitution model.

the measured dry bulk modulus. The low frequency data for the S-waves (triangles in Figure 2.16) follow closely the assumption of Gassmann’s equation of a constant shear modulus. A similar velocity dependence on heterogeneous fluid distribution was seen by Cadoret et al. (1995).

The change in S-wave velocity can be explained by the density increase with increasing saturation. The measurement for the ultrasonic frequency range stays nearly constant (squares). Since density increases with increasing brine saturation, we would
Figure 2.16. S-wave velocity in a North Sea sandstone at a differential pressure of 6.9 MPa as a function of brine saturation. Data are shown for low frequencies (triangles) and ultrasonic frequencies (squares). The dotted line shows a Gassmann substitution model.
expect the shear velocity to drop \((G = const.)\). This constant might be explained by a slow compaction within the sample. This stiffness increase may be caused by the compressing of grain contacts or closing of fractures. However, the low frequency measurements do not show any indication of increasing stiffness of the sample. The higher velocities for the ultrasonic measurements might better be explained by ever shortening of the sample caused by creep, which is not accounted for or by the dispersion effect caused by local flow [Mavko & Nur (1975) and O’Connell & Budiansky (1977)].

### 2.4.4 Results: Samples Z #5 and Z #13

Both samples, Z #5 and Z #13, are clastic sediments from the same well, 30 m apart in depth. Figures 2.17 2.18 show the P-wave and S-wave velocities for these two samples at different differential pressures. To reduce hysteresis effects, the samples were saturated with water and fluid pressure was kept constant at 3.4 MPa (500 psi). The sample was pressure-cycled twice before taking the measurements. Figures 2.17 and 2.18 show the pressure dependency of the velocities. Differential pressure is first increased from 3.4 MPa, up to 20.7 MPa, and then decreased back to 3.4 MPa.

Measurement errors are evident for sample Z #5 at 3.4 MPa; lower differential pressure should result in lower velocities under the assumption that the pore fluid pressure remains constant. Otherwise, a general trend can be observed with the increase of velocity following the increase of confining pressure for all four frequency ranges. The results of both samples were subdivided and averaged into four frequency ranges: 7.6-21 Hz, 51-201 Hz, 300-2000 Hz, and 500 kHz. A general trend of the frequency behavior emerges from Figures 2.17 and 2.18. At the same differential pressure, velocities are higher for higher frequencies. The difference between the low
Figure 2.17. Sample Z #5 P- and S-wave velocity vs. differential pressure for different frequency ranges (8-21 Hz, 50-200 Hz, 300-2000 Hz, 500 kHz), 100% water saturated at pore pressure $P_p=3.4$ MPa

frequency measurement domain and the results for ultrasonic measurements can be as large as 20% for both samples. This may be due in large part to sample heterogeneity.

Velocity dispersion is coupled to attenuation. Internal friction between the grains (Walsh, 1966), fluid motion, Biot inertial coupling (Biot, 1956) and local flow [Mavko & Nur (1975), and O'Connell & Budiansky (1977)] have been invoked as models to describe this behavior of rocks. Toksöz & Johnston (1981) give a good overview of the
Figure 2.18. Sample Z #13 P- and S-wave velocity vs. differential pressure for different frequency ranges (8-21 Hz, 50-200 Hz, 300-2000 Hz, 500 kHz), 100% water saturated at pore pressure $P_p=3.4$ MPa
definition and terminology of attenuation and quality factor. We can gain an estimate of attenuation through the coupling of velocity dispersion and quality factor \(Q\) since \(Q\) was not directly measured for these samples. One of the possible models is the Cole-Cole equation (Cole & Cole, 1941), which couples dispersion and attenuation in this case for Young’s modulus \(E\):

\[
E_{\omega} = E_{\infty} + \frac{E_0 - E_{\infty}}{1 + (i\omega\tau)^{1-\kappa}} = E_R + iE_I. \tag{2.8}
\]

With \(\tau\) being the relaxation time and \(\kappa\) is a distribution parameter, giving the range over which a relaxation mechanism operates, \(E_{\infty}\) and \(E_0\) are the infinite and zero frequency moduli, respectively, which defines our maximum dispersion. Then attenuation \((1/Q_E)\) can be described as the ratio of the imaginary and real components \((E_R\) and \(E_I)\) of a modulus \(E\):

\[
\frac{1}{Q_E} = \frac{E_I}{E_R}. \tag{2.9}
\]

Figures 2.19 and 2.20 show plots of measured Young’s modulus as a function of frequency, along with the modeled data for Young’s modulus and the quality factor \(Q_E\). A model of a single relaxation mechanism is used to fit the modulus data (Cole & Cole, 1941) and predict the quality factor. The parameters of this fitting is shown in Table 2.2. The sample was saturated with water at a fluid pressure of 3.4 MPa and a differential pressure of 6.9 MPa. Measurements from the The gray line shows the calculated quality factor of Young’s modulus, \(Q_E\). The modeled relaxation peak is at a frequency of 10 kHz. The peak for sample Z #13 is also at the same frequency. However, the amplitude of the inverse quality factor is smaller.

Figure 2.19 indicates that sample Z #5, which has higher measured permeability \((k = 27 mD)\), also has higher attenuation compared to sample Z #13 \((k = 1 mD)\).
Figure 2.19. Sample Z #5 Young’s modulus and quality factor vs. frequency, 100% water saturated at pore pressure $P_p=3.4$ MPa with a Cole-Cole fitting.

### Table 2.2. Parameters for Cole-Cole equation for samples Z #5 and Z #13

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sample Z #5</th>
<th>Sample Z #13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relaxation Peak $\tau_{[sec]}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Distribution Parameter $\alpha$</td>
<td>0.51</td>
<td>0.65</td>
</tr>
<tr>
<td>Zero Frequency Young’s Modulus $E_0 [GPa]$</td>
<td>20</td>
<td>21.7</td>
</tr>
<tr>
<td>Infinite Frequency Young’s Modulus $E_\infty [GPa]$</td>
<td>38.5</td>
<td>39</td>
</tr>
</tbody>
</table>
Figure 2.20. Sample Z #13 Young’s modulus and quality factor vs. frequency, 100% water saturated at pore pressure $P_p=3.4$ MPa with a Cole-Cole fitting.
Note that the magnitude of the attenuation peak is strongly dependent on the ultrasonic velocity. If the large difference between the ultrasonic and low-frequency velocities is due to sample inhomogeneity, then the estimated attenuation can be in substantial error.
2.5 Discussion and Conclusion

The results show that the dispersion effect cannot be neglected and can be significant even in the seismic frequency band. Comparisons between velocities acquired with seismic, logging, and in the laboratory can show large differences, especially to highly permeable rocks and when the fluid has a high modulus. Theoretical models, particularly those assuming zero or low frequencies, can fail to correctly predict the rock behavior.

We have shown that the conditions of fluid inside the rock have a large influence on the elastic properties of the rock-fluid system. If a macroscopic flow is present, at low frequencies the fluid has time to equilibrate on a large scale. In this case, the rock reaches nearly the properties of a drained system. Increasing frequency will result in only small-scale pore pressure equilibrium and a response that can be modeled with Gassmann’s equation. However, a substantial change of velocities can be obtained by increasing the frequency to ultrasonic frequencies, where the fluid pressure inside the rock is not in equilibrium. This is an unrelaxed state, where different dispersion mechanisms, like local flow or the Biot mechanism, may have a large influence. A schematic summary of the effects of these fluid-flow mechanisms on the elastic properties on a homogeneous saturated rock is shown in Figure 2.21 and summarized in Table 2.3.

As the relaxation times (frequencies) of different flow mechanisms are crossed, attenuation will peak and the elastic modulus (velocity) will increase. In reality, the relaxation times of these mechanisms will overlap and the step like modulus increase shown schematically in Figure 2.21 will smear to a more gradual and uniform increase. Obviously, because of the large impact these flow conditions can have on seismic data, more work has to be done to identify and quantify the fluid-flow mechanisms at the
<table>
<thead>
<tr>
<th>Definition</th>
<th>Condition</th>
<th>Scale of $P_p$ Equilibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drained</td>
<td>$\frac{\partial P_p}{\partial x} = 0$</td>
<td></td>
</tr>
<tr>
<td>Macro Flow</td>
<td>$P_{\text{Region}1} = P_{\text{Region}2}$</td>
<td>$\approx Wavelength$</td>
</tr>
<tr>
<td>Relaxed</td>
<td>$P_{\text{Pore}1} = P_{\text{Pore}2}$</td>
<td>$\leq Wavelength$</td>
</tr>
<tr>
<td>Gassmann Brown and Korringa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unrelaxed</td>
<td>$P_{\text{Pore}1} \neq P_{\text{Pore}2}$</td>
<td>$&lt;&lt; Wavelength$</td>
</tr>
<tr>
<td>Biot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local Flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscoelastic</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3. Systematic understanding of the frequency and pore pressure equilibration scales
Figure 2.21. Relative position of different relaxation mechanisms versus frequency, or fluid mobility different scales.

The future trend in applied geophysics is to progress from a qualitative interpretation towards a quantitative interpretation. Particularly in time lapse-seismic seismic, we must extract fluid type and saturation, as well as zones with preferred flow paths. We have seen that the elastic properties depend on the distribution of the fluid within the sample, as well as the mobility of the fluid and the frequency at which they are measured. As a result, measurements in the seismic frequency range do not necessarily correspond to the low frequency limits assumed by Gassmann (1951) or Toksöz & Johnston (1981). This is especially true for rocks with low fluid mobility such as shales, shaly sandstones, or tight sandstones. The proper flow mechanism must be used when modeling the seismic response. We will see in Chapter 5 how the use of an inappropriate model can lead to serious errors in the inverted fluid content.
Chapter 3

FREQUENCY DEPENDENT POLAR ANISOTROPY IN SHALES

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Preserved core plugs at different angles of the symmetry axis have been used to make low frequency and ultrasonic measurements on shales. Over the measured frequency range, dispersion effect can be observed for a high porosity shale in the components of the stiffness tensor. While the shear components are not very frequency dependent, the components of the stiffness tensor for compressibilities show large frequency dependent responses. The low porosity shale sample showed significant dispersion effects between the low frequency and ultrasonic frequency range.
3.1 Introduction and Elastic Properties of Shales

Over the last few years, seismic data quality has increased dramatically and is the common tool to describe the reservoir properties. Seismic reflections are sensitive only to impedance difference of two layers. We are developing an understanding of properties of one layer, sandstones, but the elastic properties of the other layer that make up the reflection above or below a reservoir, typically a shale, have to be evaluated. Little has been done to understand the seismic properties of shale. There are four main reasons for this lack of knowledge.

First, for only a qualitative or structural interpretation of seismic data, we do not need an understanding of the interface properties, as long as the interfaces can be distinguished. Secondly, shales are not reservoir rocks; this means that they are not interesting in terms of production, so they have been largely overlooked in geophysics. Also, shale cores are difficult to obtain, since the clays within shales are often sensitive to alteration. Finally, the permeability of shales are low, so measurements on shale cores are slow and time consuming.

Anisotropy in shales has broad implications in geophysics: Hornby et al. (2003) showed that deviated bore-holes have to be corrected if upscaled and used to create synthetic seismograms. Banik (1990) presented that anisotropy can be used to differentiate lithologies in the subsurface. Tsvankin (1996) and Alkhalifah et al. (1996) showed that anisotropy has significant influence in processing and interpreting seismic data.

Overall, the term “shale” is ill defined and the structure of shale can be very complex, which makes it more difficult to communicate among the geoscience disciplines. In McGraw-Hill’s Dictionary of Geology Mineralogy (Parker, 1997), a shale is defined as: "A fine-grained laminated or fissile sedimentary rock made up of silt- or claysize
particles; generally consists of about one-third quartz, one-third clay materials, and one-third miscellaneous minerals, including carbonates, iron oxides, feldspars, and organic matter.” This definition is based on the grain size of the rock and some average definition of the mineralogy. In contrast, a functional definition for many is ”any non-reservoir rock” or simply, any material with a gamma ray log response greater than 85 API.

It has been known for some time that porosity is a very important parameter in characterizing the properties of shales. The porosity and density of shales are very sensitive to stress regimes and result in substantial compaction with depth. Figure 3.1 shows common compaction curves compiled by Baldwin & Butler (1990). The porosity reduction can best be modeled with an exponential decay as a function of burial depth. Near the surface, the porosity (1-solid fraction) is typically around 60%, which is reduced to 5-10% porosity at a depth larger than 4000 m, depending on the area. Not only does stress affect shale properties, but also the mineralogical composition (Bjorlykke, 1998).

Chilingar & Knight (1960) present data of measured moisture content of three different clay minerals (montmorillonite, illite, and kaolinite), as a function of confining pressure. In Figure 3.1, illite and kaolinite show similar behavior, while montmorillonite has a much larger moisture content in comparison to the other two clay minerals at the same pressures. At high pressures, the difference in moisture content (porosity) among the probed clay minerals is decreasing; however, substantial differences between montmorillonite, and illite and kaolinite exist. Due to the layered clay structure, montmorillonite can absorb large amounts of water into its structure, which is the reason why it termed a “swelling clay”. Illite and kaolinite cannot absorb nearly as much water into their structures, which is the reason why they are known
as “non-swelling clays”.

Despite their dominant abundance in sedimentary rocks, clay properties are not well known and published results contradict each other. Wang et al. (2001) published bulk moduli for measurements of kaolinite, which ranged from 46.0-47.7 GPa. Katahara (1996) reported the bulk modulus of kaolinite as 55.5 GPa. Illite was reported by Wang et al. (2001) with 62.2 GPa and by Katahara (1996) with 52.3 GPa. The differences can be explained by the uncertainties of obtaining the results in both studies: Katahara’s values derived from published mineral analog data, while Wang measured elastic properties of epoxy and clay mixtures. These numbers are in huge disagreement with the published data of Woeber et al. (1963), where he found a bulk modulus of kaolinite of only 1.5 GPa. This value of kaolinite is even less than that of pure water (2.2 GPa at room conditions). This raises the question of whether bound water in clay minerals should be considered part of the mineral structure; further investigation is needed.

Several different possibilities exist to model the elastic properties of shales. Most models are based on representing clays as planar building blocks of shale that get aligned with compaction, e.g. Hornby et al. (1994), Sayers (1999), Jakobsen et al. (2003), Johansen et al. (2004). This arrangement will cause strong anisotropy. Sayers (2005) discussed the concept of the alignment of these clay platelets. This idea was introduced by Terzaghi (1956), who called it “clay clusters”. Later, Aylmore & Quirk (1960) named it “domains”. Rundle & Schuler (1981) modeled shale properties as an isotropic background with inclusions.

If properties of shales are measured and discussed, anisotropy has to be a focus. Kaarsberg (1959) was one of the first to measure shale anisotropic properties. Jones & Wang (1981), Vernik & Nur (1992), and Hornby et al. (1994) published a few mea-
Figure 3.1. Top: Global compaction trends for shales (Baldwin & Butler, 1990), porosity vs. depth. Bottom: Compaction of kaolinite, illite and montmorillonite (Chilingar & Knight, 1960) porosity vs. pressure.
surements on shale. Few comprehensive studies have been conducted and published discussing anisotropy in shales. Wang (2002a,b) published data from 16 shale samples, which shows a strong correlation between P-wave and S-wave anisotropy. He showed that anisotropy decreases with increased pressure and increased porosity.

A comparison of Wang’s data with field studies (Sarkar & Tsvankin, 2004) showed anisotropic parameters are different at different frequencies. These variations are usually explained by the different measurement scales. Anisotropy can also be induced by the layering caused by deposition, compaction, and diagenesis. Hauge (1981) measured attenuation in different wells with varying lithologies, velocities, and attenuations; he also observed that the attenuation for shales is very small in comparison to sands. He also observed that the attenuation seems to be independent of the compaction (depth) of the shale.

Klimentos & McCann (1990) came to different conclusions. They showed that the ultrasonic attenuation of clay rich sandstone samples (between 0-30% clay content) is mainly dependent on the how much clay is in the sandstone and not on porosity. For the high clay content they observed a $Q_P$ at 1 MHz of about 10. Stanley & Christensen (2000) measured velocities and attenuation parallel and perpendicular to the bedding planes of four shale samples. They concluded that if the shales are saturated and at high differential pressure, velocity anisotropy in shales can be observed, but no substantial attenuation anisotropy was measured.

Only a few studies have been done comparing the elastic properties over a large frequency range. White et al. (1983) observed that P-wave velocity measurements in the Pierre Shale from sonic logs was consistently higher (an average of 6%), than velocities derived from VSPs. Hornby et al. (1995) showed that in sands, the difference between measurements done with a sonic tool (10 kHz) and core measurements
(1 MHz) were only 3%. For shales, however, this velocity difference is much larger, up to 15%, with the P-wave velocity being much faster in the ultrasonic measurements in comparison to sonic measurements. Suarez-Rivera et al. (2001) presented a case study of shale measurements for the Pierre Shale at different frequencies. P-wave velocity dispersion was observed and reached as much as 45% between the low frequency measurements (10 Hz) and ultrasonic measurements (500 kHz). It is unknown what caused these dispersion effects; however, one explanation may be the different strain level for different measurement techniques.

Another well documented example to compare elastic properties at different frequencies is documented by Schön (1996). A homogeneous section of the Continental Deep Drilling Project (KTB) well has been used to extract anisotropic parameters from VSP and core samples measured in the laboratory at ultrasonic frequencies. The results for the derived stiffness tensors are shown in Figure 3.2. On the left side of the figure, all five independent components of the stiffness tensors, $C_{ij}$’s, are plotted as a function of frequency. Increasing frequency results in an increase of $C_{11}$ and a decrease in $C_{33}$, while the other components of the stiffness tensor remain nearly constant. As a result, the Thomsen parameters (left) show a frequency (or scale) dependent behavior.

The parameter for the compressional waves ($\epsilon$) shows a large increase from 2% to 15%. As a result, $\delta$ also changes from negative to positive. One explanation for the difference can be the sample selection. Micro fractures can be induced and may influence the measured anisotropy. Also, questions about the effect of layering and stress induced anisotropy must be answered (Prioul et al., 2004). Gao (2003) shows how stress can cause a self-organization to minimize the combined free energy, which leads alignment and to anisotropy.
3.2 Polar Anisotropy and Gage Mounting

3.2.1 Measurement Setup versus Polar Anisotropy

The measurement principle for low frequency measurements are based on Spencer (1981) and has been described by Batzle et al. (2006). A sinusoidal stress is applied to the cylindrical sample and the deformation is measured by strain gages mounted horizontally and vertically on the rock sample and vertically on a standard (Aluminum). If the rock is isotropic and homogeneous, these two measurements are enough to calculate two independent elastic properties of the rock. Hooke’s law connects strain $\epsilon_i$ with stress $\sigma_j$ through the compliance tensor $S_{ij}$:

$$\epsilon_i = S_{ij}\sigma_j.$$  \hfill (3.1)
In the case of a polar or transversely isotropic symmetry, the rock has five independent elastic properties. The general form is:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\
S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}.
\] (3.2)

Using the low frequency setup, we are confronted with a challenge: the apparatus can only apply a sinusoidal stress in the \(\sigma_3\) direction (Figure 3.3). A static hydrostatic stress is imposed equally in the directions of \(\sigma_1\), \(\sigma_2\), and \(\sigma_3\). No direct shear stresses, \(\sigma_4\), \(\sigma_5\), and \(\sigma_6\), can be applied and no shear strain gages are attached to the sample in the current configuration. So, the previously introduced equation would reduce to:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
0 \\
0 \\
0 
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\
S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
\sigma_3 \\
0 \\
0 \\
0
\end{bmatrix}.
\] (3.3)

The solution to this problem is to use core plugs and mounted gages at different angles. Exadaktylos (2001) and Valesa et al. (2004) showed that the static elastic properties for a transversely isotropic material can be derived from strain
Figure 3.3. Indication for direction of the cored plug for the measurements

\[ \sigma_1 = 0 \]

\[ \sigma_2 = 0 \]
measurements at different angles. By using a Bond transformation [Auld (1973) and Winterstein (1990)], the response of strains for core plugs at different angles can be predicted.

At first, the compliance tensor \( S_{ij} \) has to be rotated into the direction of the core symmetry as \( S'_{ij} \):

\[
S'_{ij} = N S_{ij} N^T, \tag{3.4}
\]

by using the rotation matrix \( a \) with a rotation the core plug of an angle \( \theta \) around \( x_1 \) axis:

\[
a_{x_1} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}, \tag{3.5}
\]

which results in the Bond rotation matrix \( N \) for the \( x_1 \) direction:

\[
N_{x_1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & c^2 & s^2 & cs & 0 \\
0 & s^2 & c^2 & -cs & 0 \\
0 & -2cs & 2cs & c^2 - s^2 & 0 \\
0 & 0 & 0 & 0 & c -s \\
0 & 0 & 0 & 0 & s & c
\end{bmatrix}. \tag{3.6}
\]

Here \( c = \cos \theta \) and \( s = \sin \theta \) with the rotation angle \( \theta \). A zero degree rotation angle means that the bedding planes in the shale core are horizontal; a 90° angle indicates vertical bedding planes, as shown in Figure 3.4.

If we use a known compliance tensor of one of the shales (Table 3.1), we can calculate how strain will change as a function of the rotation of the compliance tensor around the \( x_1 \) axis by applying a stress of \( \sigma_3 = 14kPa (= 14 \times 10^{-6}GPa = 2psi) \). The
Figure 3.4. Direction of the coring angle ($\theta$) for the shale plugs.
results of this computation are shown in Figure 3.5.

The strain in \( x_3 \) direction of a horizontal core plug (\( \theta = 0^\circ \)) will be directly related to \( S_{33} \). At the other extreme, after a rotation of \( \theta = 90^\circ \), the strain in \( x_3 \) is directly related to \( S_{11} \).

Not only the direction of the core plug can be chosen, but the direction of the mounted strain gages also can be varied. Again, a Bond transformation has to be applied to rotate the strain field:

\[
\epsilon' = \epsilon N.
\] (3.7)

The result of the rotation of the strain measurement is shown in Figure 3.7. First, the direction of the rotation around the \( x_1 \) axis is shown in Figure 3.6, G1 and G2 indicate the two main strain gage directions, which are rotated by an angle \( \alpha \). Next, the resulting strain after rotation around \( \alpha \) is shown, where \( \epsilon_5 \) and \( \epsilon_6 \) are zero, while \( \epsilon_4 \) is strongly dependent on the rotation angle. Since we rotate around the \( x_1 \) axis, \( \epsilon_1 \) is not affected by the rotation. Both \( \epsilon_2 \) and \( \epsilon_3 \) show expected behavior: \( \epsilon_2 \) at \( \alpha = 0^\circ \) is equal to \( \epsilon_3 \) at a rotation angle of \( \alpha = 90^\circ \) and vice versa. Note that if the rotation angle is \( \alpha = 26^\circ \), \( \epsilon_2 = 0 \) and if the rotation angle is \( \alpha = 64^\circ \), \( \epsilon_3 = 0 \).

<table>
<thead>
<tr>
<th>( S_{ij} )</th>
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</thead>
<tbody>
<tr>
<td>( S_{11} )</td>
<td>0.0988</td>
</tr>
<tr>
<td>( S_{33} )</td>
<td>0.1743</td>
</tr>
<tr>
<td>( S_{12} )</td>
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</tr>
<tr>
<td>( S_{13} )</td>
<td>-0.0238</td>
</tr>
<tr>
<td>( S_{44} )</td>
<td>0.3745</td>
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<tr>
<td>( S_{66} )</td>
<td>0.2392</td>
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Table 3.1. Shale compliances to calculate the strain response for the rotation of the sample and strain gages.
Figure 3.5. Calculated strain as a function of coring direction ($\theta$) of a shale plug $\theta = 0^\circ$ to $\theta = 90^\circ$ around the $x1$ axis, using compliance tensor 3.1.
Figure 3.6. Definition of the strain gages rotation angle ($\alpha$)
Figure 3.7. Computed strain response for the rotation angle $\alpha = 0^\circ$ to $\alpha = 90^\circ$ for an horizontal core ($\theta = 0^\circ$) around the $x1$ axis, using compliance tensor 3.1
The same method to calculate the strain gage response can be applied to core
plugs of 45° and 90°, shown in Figure 3.8. By comparing the maximum strain for
the 0° core and the 90° core, we can see that $\epsilon_{3}^{0^\circ} > \epsilon_{3}^{90^\circ}$ under the condition of gages
rotation of $\alpha = 0^\circ$. These findings are summarized in Table 3.2, where strain and
compliances for different core ($\theta$) and strain gage ($\alpha$) orientation also are shown.
Figure 3.8. Definition of the strain gages rotation angle (α) and the strain response for the rotation angle $\alpha = 0^\circ$ to $\alpha = 90^\circ$ for an horizontal core (top - $\theta = 45^\circ$, bottom - $\theta = 90^\circ$) around the $x_1$ axis, using compliance tensor 3.1.
By using only cores oriented at $\theta = 0^\circ$ and $\theta = 90^\circ$ we can resolve four out of five of the unknown compliance components, $S_{33}$ and $S_{13}$ from the $\theta = 0^\circ$ core plug, and $S_{11}$, $S_{12}$ and $S_{13}$ from the $\theta = 90^\circ$ core plug. By adding measurements on a plug at $\theta = 45^\circ$, then $S_{33}$ and $S_{44}$ can be resolved. Thus, with several plugs, all five unknowns can be measured. For example:

\[
S_{11} = \frac{\epsilon_3}{\sigma_3}{\bigg|}_{\theta=90^\circ, \alpha=0^\circ} \\
S_{12} = \frac{\epsilon_1}{\sigma_3}{\bigg|}_{\theta=90^\circ, \alpha=0^\circ} \\
S_{13} = \frac{\epsilon_2}{\sigma_3}{\bigg|}_{\theta=90^\circ, \alpha=0^\circ} \\
S_{33} = \frac{4\epsilon_2}{\sigma_3}{\bigg|}_{\theta=45^\circ, \alpha=0^\circ} + S_{44} - S_{11} - 2S_{13} \quad (3.8) \\
S_{44} = \frac{4\epsilon_3}{\sigma_3}{\bigg|}_{\theta=45^\circ, \alpha=0^\circ} - S_{33} - S_{11} - 2S_{13} \\
S_{66} = 2(S_{11} + S_{12})
\]
<table>
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<tr>
<th></th>
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<td>$1/4(S_{11}+2S_{13}+S_{33}+S_{44})$</td>
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<tr>
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<tr>
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<td>$1/2(S_{13}+S_{11})$</td>
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<td>$0$</td>
<td>$0$</td>
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<td>$0$</td>
</tr>
<tr>
<td>$\epsilon_6$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 3.2. Relations between strain and compliances for different core ($\theta$) and strain gage ($\alpha$) orientations
3.2.2 Gage and Sample Mounting

A schematic of a prepared sample is shown in Figure 3.9, a layers of polyimide film (Kapton®) is first attached to the core plug with epoxy (K20). Next, the strain gages are glued to the exterior to pressure seal the sample. Since nitrogen gas is used to apply confining pressure, sealing the sample has to be done very carefully to avoid gas leakage inside the sample. This will change the saturation from 100% to partially saturated. The aluminum end-pieces with attached strain gages are used as load-cells to measure the applied stress on the sample.

Figures 3.10 - 3.12 illustrate the different stages of sample preparation. Figure 3.10 shows the three cut core plugs from preserved cores. After the core plugs are cut and flattened, the next step is to glue both aluminum end pieces to the plug (Figure 3.11) and coat the sample with a very thin layer of epoxy (K20) to avoid drying out the sample. It is important to flatten the ends of the core and make them as parallel as possible to allow a uniform stress distribution across the sample when measured. Attaching the Kapton® film is shown in Figure 3.12 (left). Finally, the strain gages are glued to the sample at the desired orientations. As previously described, we have modeled the directions the gages must be attached to the sample. Figure 3.13 shows one possible setup of the gage location on samples of $\theta = 45^\circ$ and $\theta = 90^\circ$. 
Figure 3.9. Schematic sample setup for the low frequency (LFQ) measurements, same as Figure 2.4.
Figure 3.10. Cored Shale 1 samples for $0^\circ$, $45^\circ$, and $90^\circ$; sample diameter is 3.8cm

Figure 3.11. Sample preparation for $90^\circ$ Shale 1 sample
Figure 3.12. Sample preparation for 45° Shale 1 sample

Figure 3.13. Strain gage mounting for a core plug $\theta = 45^\circ$ (left) and $\theta = 90^\circ$ (right)
3.3 Measurements

3.3.1 Sample Characterization

Two shale samples have been investigated in this study to better understand the effect of porosity on the elastic properties. One set of samples with high porosity (Shale 1: 25.5%) and one set with very low porosity (Shale 2: 7.5%) were chosen. Mercury injection porosimetry was used to characterize the pore space of these samples: from the injection pressure \((P_{\text{inj}})\) of the non-wetting fluid phase, the radii \((r)\) of pore throats can be calculated:

\[
r = \frac{2\Gamma \sin \psi}{P_{\text{inj}}},
\]

where \(\Gamma\) is the surface tension of the mercury and \(\psi\) is the contact angle between the mercury and the rock forming solid.

Figure 3.14 shows the fraction of bulk volume of the rock (top) and pore space (bottom) occupied by mercury as function of the pore throat radius. The total porosity of the rock can be obtained directly by the injection profile. The two shales show substantial differences: first, the total porosity is three times as large for Shale 1 compared with Shale 2 (25.5% - 7.5% respectively). Second, the maximum change in mercury saturation is at a much larger pore throat radius for Shale 1 (13.3 nm) compared to a pore throat radius of Shale 2 (4 nm). The interpretation of the pore throat size distribution deserves more investigation: the examination of scanning electron microscope (SEM) images (Figure 3.16) shows an elliptical pore shape and not round capillares. Boltona \textit{et al.} (2000) showed that the pore size distribution can be anisotropic, but this has not been taken into account in this study of the pore throat distribution. While the distribution of the porosity fraction (Figure 3.14)
shows a sharp change for Shale 1, the distribution for Shale 2 is much broader. This behavior of the pore throat size distribution can be explained by different compaction of the shale samples.

Figures 3.15 and 3.16 show SEM pictures of both shales for an amplification of 100x and 1200x. Pore space, silt, and sand grains are visible, and it can be seen that the pore diameter is small for the Shale 2. At large amplifications, the so-called “domains” or “clusters” are visible, and these consist of stacks/booklets of clay minerals.

In addition to the porosity and pore size distribution, the mineralogical composition was acquired to understand if different minerals are present. XRD-analysis (Suryanarayana & Norton, 1998) was done and the results are shown in Table 3.3.

Surprisingly, the two shales have nearly identical composition (Talbot, 2005); the amount of quartz (29.6% and 28.5%) is equal for both samples, as well as the amount of siderite, kaolinite, and chlorite. The only differences that exist are the amount of illite and randomly ordered mixed-layer illite/smectite (R=0 M-L I/S). Shale 1 consists of more illite and less mixed-layer illite/smectite; the opposite is true for Shale 2. Since XRD-analysis is semiquantitative, overall the mineralogy of both samples can be assumed as very similar.
Figure 3.14. Mercury injection results: top - bulk volume fraction saturated with mercury as a function of pore throat size, ranging from 1nm to 1µm, bulk volume fraction at 2nm corresponds with effective porosity; bottom - pore volume fraction saturated with mercury.
Figure 3.15. Scanning Electron Microscope pictures from Shale 1 (left) and Shale 2 (right) for a magnification of 100x. Q-quartz, P-pore space

Figure 3.16. Scanning Electron Microscope pictures from Shale 1 (left) and Shale 2 (right) for a magnification of 1200x. Note the strong mineral orientation for both samples. Q-quartz, C-clay plates
Table 3.3. Mineralogical Composition of Shale 1 and Shale 2 (Talbot, 2005)

<table>
<thead>
<tr>
<th>Mineral</th>
<th>SHALE 1</th>
<th>SHALE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>0.296</td>
<td>0.285</td>
</tr>
<tr>
<td>K-Feldspar</td>
<td>0.031</td>
<td>0.054</td>
</tr>
<tr>
<td>Plagioclase</td>
<td>0.030</td>
<td>0.034</td>
</tr>
<tr>
<td>Siderite</td>
<td>0.113</td>
<td>0.115</td>
</tr>
<tr>
<td>Smectite</td>
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<td>0.000</td>
</tr>
<tr>
<td>R=0 M-L I/S*</td>
<td>0.062</td>
<td>0.134</td>
</tr>
<tr>
<td>R=1 M-L I/S**</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>R=3 M-L I/S***</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Illite and Mica</td>
<td>0.142</td>
<td>0.043</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>0.248</td>
<td>0.242</td>
</tr>
<tr>
<td>Chlorite</td>
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<td>0.074</td>
</tr>
<tr>
<td><strong>Total</strong></td>
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<td>1.000</td>
</tr>
</tbody>
</table>

3.3.2 Results

To better ensure pore pressure equilibration in these low permeability samples, pressure changes must be done slowly. Initially, the sample was placed under confining pressure with no pore pressure. High saline brine (19% KCl) was then injected on both sides to completely saturate the sample and prevent the ends of the sample from drying out. After the sample equilibrated, the next pressure step was applied. The estimated time for the sample to equilibrate was one day (e.g., one day is needed for a pore pressure pulse to be transmitted through the sample). Figure 3.17 shows the pressure schedule. For the first five days (120 hours), no pore pressure was applied. The differential pressure (diamonds) increases, due to the increase in confining pressure (squares), while the pore pressure (triangles) remains constant. Each symbol indicates one set of measurements, ranging from 3-3000 Hz for the low frequency measurements and 800 kHz for the measurements at ultrasonic frequencies. An example
Figure 3.17. Confining and pore pressure as function of time for core Shale 1, $\theta = 45^\circ$: differential pressure (diamonds), confining pressure (squares) and pore pressure (triangles) are plotted.
Figure 3.18. P-wave travel time as function of time (days) after a change in stress of the time dependent P-wave travel time changes is shown in Figure 3.18. After 1.5 days the travel time did not change anymore, the velocity was constant.

An example of the measured strain in Shale 1 (core $\theta = 45^\circ$) is shown in Figure 3.19. (The arrangements of the strain gages are shown in Figure 3.13; Y1 and Y2 are strain measurements for $\epsilon_3$; P1, P2, and P3 measure a combination of $\epsilon_1$ and $\epsilon_2$.) From the strain measured with the strain gage on the Aluminum load-cell, the applied stress could be ascertained and used to calculate the compliances.
Figure 3.19. Measured strain (left) and applied stress (right) for Shale 1 core $\theta = 45^\circ$.

By applying the analysis described above, from the measured strain ratios and absolute strains, the compliance tensor can be derived (Figure 3.20) and inverted to the stiffness tensor, as shown in Figures 3.21 and 3.22. Shale 2 is much stiffer than Shale 1; $C_{11}$ and $C_{44}$ for Shale 2 are four times greater. While the stiffness is nearly independent of frequency for Shale 2 ($\phi = 7.5\%$), the stiffness of Shale 1 ($\phi = 25.5\%$) shows some interesting differences. Some of the components of the stiffnesses are nearly frequency independent; others show a large dispersion effect: $C_{11}$ and $C_{33}$ (compressional components) show a large frequency dependency, whereas $C_{44}$ and $C_{66}$, (shear components) show nearly no frequency dependence. The stiffness $C_{11}$ increases from 12 GPa to 20 GPa, while $C_{33}$ increase from 8 GPa to 13 Gpa as function of frequency. The shear stiffness $C_{66}$ increases from 4 to 5.5 Gpa, $C_{66}$ from 3 GPa to 3.8 GPa. Since the off-diagonal components of the stiffness tensors are a combination of normal and shear, $C_{12}$ and $C_{13}$ also show significant dispersion effects, changing from 2 GPa and 3 GPa to 8 GPa and 8.5 GPa, respectively.

From stiffness and densities, the P-wave and S-wave velocities and Thomsen
Figure 3.20. Calculated compliances for Shale 1 (left) and Shale 2 (right) as a function of frequency from 3 Hz to 800 kHz at the in-situ differential pressures parameters can be calculated as shown in Figures 3.23 and 3.24. The velocities show the expected behavior, \( v_{P0} < v_{P45} < v_{P90} \) and \( v_{S_{slow}} < v_{S_{fast}} \).

While for high/ultrasonic frequencies the values for the \( \epsilon \) and \( \gamma \) parameters are between 0.2 and 0.3, the values for seismic frequencies are slightly higher. However, the \( \delta \) parameter shows a substantial difference between low/seismic and ultrasonic frequencies, close to zero for low frequencies, 0.2 (Shale 1) and 0.05 (Shale 2). Thus, the Thomsen parameters are frequency dependent and discrepancies reported previously cannot all be attributed to scale or different measurement techniques. The significance of this frequency dependence will be one area of our future research.
Figure 3.21. Calculated stiffness tensor for Shale 1 as a function of frequency from 3 Hz to 800 kHz at the in-situ differential pressures
Figure 3.22. Calculated stiffness tensor Shale 2 as a function of frequency from 3 Hz to 800 kHz at the in-situ differential pressures
Figure 3.23. Calculated Velocities for Shale 1 (left) and Shale 2 (right) as a function of frequency from 3 Hz to 800 kHz at the in-situ differential pressures

Figure 3.24. Thomsen parameters [ε, δ, and γ, Thomsen (1986)] for Shale 1 (left) and Shale 2 (right) as a function of frequency from 3 Hz to 800 kHz at the in-situ differential pressures
3.4 Conclusions

Both shale samples have a similar mineralogical composition, but show a large difference in porosity, in the amount and the pore throat diameter. Differences in anisotropy parameters as a function of frequency cannot simply be caused by the effect of layering, but point towards another frequency dependent mechanisms. Two plugs from the same preserved core have been used to measure the five independent elastic constants of a shale. The measurements from both plugs had to be combined to obtain all independent components. A substantial dispersion effect for some of the elastic properties has been observed, while other components of the stiffness tensor are nearly frequency independent, from seismic frequencies to the ultrasonic frequency range. Since only the compressional compliance is frequency dependent for the high porosity sample and not for the low porosity sample, we can speculate that fluid and fluid motion probably cause this difference.

The differences between the low frequency measurements and the ultrasonic measurements for Shale 1 have been validified with other measurements shown by Duranti et al. (2005). Figures 3.25 and 3.26 show the low frequency (LFQ) measurements and ultrasonic measurements in comparison to seismic measurements (VSP), well log measurements (sonic), and nano-indentation measurements. Increasing frequency results in increasing stiffness or velocities; no non-causal behavior (higher frequency, lower velocity) has been observed.

More measurements have to be done to verify the results and ascertain the physical significance of these results. Understanding the intrinsic frequency dependent shale properties could help to compare ultrasonic, well log, and seismic measurements, as well as to understand how to decouple the intrinsic, stress, and scaling induced anisotropy.
Figure 3.25. Left: Calculated velocities for several measurement techniques (LFQ, VSP, sonic logging and ultrasonics) as function of frequency, (Duranti et al., 2005)
Figure 3.26. Calculated composite modulus $M$ to compare LFQ and ultrasonic measurements to micro indentation measurements, (Duranti et al., 2005)
As a next step, attenuation measurements for these samples should be done and compared with the observed dispersion. Future research should strive to determine what is the influence of the mineralogy of the shales, the properties of the bound water, and the fundamental cause of anisotropy.
Dispersion (frequency dependent velocity) and attenuation ($1/Q$, inverse of Quality factor) are fundamental rock properties for fluid saturated rocks. As the quality of geophysical data increases, our ability to extract intrinsic $1/Q$ and dispersions improves. Potential applications for these properties include estimating fluid saturation, identifying zones of high permeability, and monitoring time-lapse surveys. However, to be more quantitative, we must understand the mechanisms involved.

Pore fluid motion is not the only relaxation mechanism responsible for observed attenuation and dispersion in seismic and well logging data. Alternative loss mechanism are for example: viscous loss within heavy oils or scattering. However, fluid mobility probably controls the magnitude and frequency of the intrinsic attenuation and the amount of dispersion in most reservoir rocks.

We show a methodology to derive velocities, reflection coefficients, and attenuation as a function of frequency from well log data. It should be possible to predict velocity dispersion and zones of high attenuation between the seismic and well logging frequency range.
4.1 Introduction

Currently, the interpretations are primarily empirical and results are ambiguous or even contradictory. As we continue to make low (seismic) frequency measurements, new mechanisms appear that produce dispersion and attenuation, which are coupled. An example of frequency dependent elastic properties is shown in Figure 4.1, which shows the measured P-wave and S-wave velocities as function of frequency for 100% brine saturation and a fluid saturation with 66% brine and 34% butane. While the S-wave velocity does not change between fully saturated with brine and saturated with brine and butane, the P-wave velocity shows a significant difference between these two saturations. At first the P-wave velocity drops after butane is injected into the core plug; the difference between these two measurements (brine and brine+butane) also increases with decreasing frequency. At ultrasonic frequencies (500 kHz) the difference is 3%; at 5 Hz this difference reaches 20%. Since the rock did not change, the change in velocity as a function of frequency, so-called dispersion, is caused by the movement of the pore fluids inside the rock sample.

With more data available, it became apparent that a host of flow mechanisms could be combined to influence elastic wave velocities and attenuations over a broad frequency range (Figure 4.2). At the extreme low frequency end, the rock will behave “drained”, or with no pore pressure build-up possible during any rock deformation. This is usually the case in “static” laboratory stress-strain measurements. Macroscopic flow between regions of different saturation or compressibility can occur at slightly higher frequencies White (1975) and Pride et al. (2004).

Intermediate frequencies give the relaxed pore pressure between adjacent pores, justifying Gassmann’s assumptions (Gassmann, 1951). At higher frequencies, local flow between pores may dominate [O’Connell & Budiansky (1977) and Mavko &
Figure 4.1. Rim Sandstone brine and partial saturated (34% butane) (Batzle et al., 2003)
Figure 4.2. Relative position of different relaxation mechanisms versus frequency or fluid mobility

Nur (1979)]. At still higher frequencies we would find mechanisms such as Biot inertial coupling (Biot, 1956), grain contact friction [Savage (1966) and Dieterich (1972)], and viscoelastic losses [O’Connell & Budiansky (1978) and Lesueur et al. (1996)]. Other relaxation mechanisms, like velocity dispersion within liquids, have been observed, e.g., by Rogez & Bader (1984), but they are not included in the concept. These processes are too complex to model over the whole frequency band. As a simplification, we used an earlier conceptual model to be able to predict velocity and dispersion. Figure 4.3 shows a schematic of the attenuation-velocity curves for a single
simple loss mechanism. One example of such a loss mechanism is macroscopic flow. In this case, pore fluid mobility, the ratio of rock permeability and fluid viscosity, is the prime factor controlling losses within the seismic band. The measurement frequency window is restricted, and for any particular rock/fluid combination, dispersion and peak attenuation could range widely over the frequency band, depending on fluid mobility. For example, in shales we can expect higher frequencies (as pictured in Figure 4.3), because of the low mobility conditions. On the other hand, in porous and permeable sands, the measurement window would be on the left, or on the low frequency side of the relaxation peak.

As mentioned before, attenuation and dispersion are coupled, which leads to the frequency dependent elastic properties. As a result, the reflection series of the Earth, as represented by a well log, has to be considered, depending on frequency. This mechanism is different than the typical geometric frequency dependence, scale dependence, like Backus averaging (Backus, 1962)
Figure 4.3. Schematic diagram showing the expected velocity/attenuation relations for a fluid mobility controlled relaxation mechanism.
4.2 Derivation

Well logs provide a large amount of information, which can be used to develop and parameterize an estimates of dispersion and attenuation. As a result, a reflection coefficient as function of frequency can be calculated. In this way, forward models of viscoelastic wave propagation can be run to produce synthetic seismic traces.

We use a well-known set of relations by Cole & Cole (1941) and applied to elastic properties by Casula & Carcione (1992) and Toverud & Ursin (2005). The Cole-Cole equation relates dispersion and attenuation, as indicated by a real \( M_R \) and imaginary \( M_I \) component of a modulus:

\[
M(\omega) = M_\infty + \frac{M_0 - M_\infty}{1 + (i\omega\tau)^{1-\kappa}} = M_R(\omega) + iM_I(\omega),
\]  

(4.1)

with \( \tau \) being the relaxation time and \( \kappa \) a distribution parameter giving the range over which a relaxation mechanism operates. \( M_\infty \) and \( M_0 \) are the infinite and zero frequency moduli, respectively, which defines our maximum dispersion. Then quality factor \( (1/Q_M) \), which is related to attenuation \( (\alpha) \) can be described as the ratio of the imaginary to real components:

\[
\frac{1}{Q_M} = \frac{M_I}{M_R} = \frac{\alpha v}{\pi f}.
\]  

(4.2)

Figure 4.4 shows the velocities and attenuations that result from different values of \( M_\infty \) and \( M_0 \). As the difference between these moduli increases, the magnitude of the attenuation increases.

From a log, for example, the zero frequency modulus can be derived as the 'dry' or drained modulus, either extracted from gas zone or calculated using Gassmann’s equations (Gassmann, 1951). Relaxation time \( \tau \) depends on the specific mechanism,
Figure 4.4. Influence of the difference between the zero and infinite frequency moduli on total dispersion and attenuation. Other parameters are fixed.
such as flow between the zones of patchy saturation. Following White (1975), we can derive a time scale or frequency at which this relaxation occurs:

\[ \tau = \frac{x^2}{4D}. \] (4.3)

Here, \( x \) is the relaxation distance and \( D \) the diffusion coefficient. As previously mentioned, mobility \( m \) of the fluid within the rock is determined by the permeability, \( k \), and viscosity, \( \eta \):

\[ m = \frac{k}{\eta}. \] (4.4)

The mobility is used with the fluid bulk modulus \( (K_{fl}) \) to derive the diffusion coefficient (White (1975) and Gist (1994)):

\[ D = \frac{k}{\eta}K_{fl} \] (4.5)

As Batzle et al. (2006) mentioned, Figure 4.5 shows that changes in permeability of the rock can have substantial affect frequency at which the peak attenuation can occur. We can model this effect by shifting relation times (as shown in Figure 4.6).

As we have seen in measurements in heavy oils (Gautam, 2003b), there may not a simple single relaxation time. In the Cole-Cole relations, this can be addressed by changing the distribution parameter. This parameter, \( \kappa \), can distribute the mechanisms over a broad frequency range, which can be related to rock or fluid property distributions causing this dispersion. The result is a smoothing of the dispersion curve and broadening of the attenuation peak, as seen in Figure 4.7.
Figure 4.5. Measured compressional velocity as a function of frequency at two pore fluid salinities and at two differential pressures ($P_d$). At high salinity, the permeability remains relatively high, significant dispersion is observed within the seismic band. At low salinity (distilled water), the low permeability and low fluid mobility force the dispersive region to lower frequencies, below our measurement window (Batzle et al., 2006).
Figure 4.6. Relaxation time ($\tau$ [sec]) shifts the attenuation and dispersion to higher and lower frequencies. For fluid flow mechanisms, it is related to the fluid mobility.
Figure 4.7. Influence of varying the distribution parameter ($\kappa$) on the dispersion and attenuation curves.
4.3 Log Responses for Field Example GOM

We can apply the previous derived relationships to develop a log of parameters that would describe the frequency dependent viscoelastic properties at depth. As an example, Figure 4.8 shows an analog outcrop adjacent to a sample well log from the Gulf of Mexico. The gamma-ray log is used as a lithology indicator, high values correspond with shales, while low gamma-ray values indicate clean sands. Next, the induction log, measuring the resistivity of the formation is used to define sections with hydrocarbon saturation. Finally, density and sonic logs are used to identify if oil or gas is present in the hydrocarbon zone.

Figure 4.9 shows the measured well logs as a function of depth. The first column plots the gamma-ray response, which indicates two sand bodies between 1500-1620 m. The first sand is between 1515-1560 m, while the second sand ranges from 1587-1600 m. Next, the water saturation is plotted, which is derived by using the induction/resistivity log. As a result, it can be observed that the upper sand between 1515-1530 m has a high hydrocarbon saturation of up to 90%. The P-wave and density log indicate that the hydrocarbon is gas. Below 1530 m the sand is brine saturated. The second sand below, has only a very low gas saturation of 10% in the upper part 1587-1593 m, as indicated by the sharp decrease in P-wave velocity from 2.35 km/s to 1.85 km/s. Again, the lower part of the sand (1593-1600 m) is brine saturated. Most models for fluid in rocks include the permeability as an input parameter. Here, no direct measure of the permeability like Stonley permeability or NMR measurements have been available, so Timur’s equation (Timur, 1968), which correlates porosity ($\phi$) and permeability ($k$), has been used to estimate the permeability, $k$:

$$k = A\phi^B.$$  \hspace{1cm} (4.6)
Figure 4.8. Outcrop example (Brushy Canyon Formation) and well log example (Gulf of Mexico) indicate changes in lithology.
$A$ and $B$ are empirical constants derived from core measurements or from well log correlations in wells with permeability measurements. In our model, a modified Timur equation is used to account for changes in the clay content ($X_{clay}$) within the section:

$$k = A\phi^B X_{clay}^D$$

(4.7)

with $A = 20000$, $B = 3$, and $D = 4$.

By using the P-wave and S-wave velocities and the previously discussed properties, all four input parameters for the Cole-Cole-equation can be estimated. The most straightforward parameters to develop are the zero and infinite frequency moduli. In the extreme, these would correspond to the drained (dry) modulus and the ultrasonic modulus. However, from the log, the difference in moduli would be determined if any two zones of identical lithology were immediately adjacent one gas saturated and one brine saturated (such as above and below 1530 meters in Figure 4.9). The relaxation time ($\tau$) in this case would be a function of the distance from the gas/brine interface. This is set by Equation 4.3, defining the time it takes for a pressure pulse to diffuse across the boundary. The most problematic of the parameters is the distribution parameter, $\kappa$. If there is a single mechanism dominating, such as diffusion to a gas/brine interface, $\kappa$ could be small. On the other hand, for heterogeneous rocks or fluid saturations, $\kappa$ could be large (close to 1) and result in a very broad frequency response. This distribution parameter requires considerably more research to understand its behavior.

In the shale above 1515 m, there can be significant separation of the zero and infinite frequency moduli, resulting in substantial attenuation. However, if we consider only bulk fluid flow, the relaxation time is too long, much below seismic frequencies as seen in Figure 4.10 (other mechanisms may be applicable). The behavior is the
Figure 4.9. Cole-Cole dispersion/attenuation parameters derived for a Gulf of Mexico log. This type of log could be used to derive a synthetic trace based on viscoelastic propagation and show frequency effects on $1/Q$. Permeability is derived from lithology and porosity logs using Timur (1968) relation.
same for the shales from 1564-1588 m and below 1600 m. At 1530 m, we enter a predominantly gas-saturated sand. Even the infinite frequency moduli will be low, so the overall attenuation is limited. In direct contact with this gas zone is a brine-saturated zone. Here the modulus separation is larger and the frequency is controlled by the diffusion distance to the gas-brine interface. A small shale break at 1545 m would prevent any such pressure diffusion. In the brine-saturated sand below 1545 m, long relaxation times would lower the observed attenuation in the seismic frequency band. In the partially gas-saturated sand between 1588 m and 1600 m, we would expect a maximum attenuation over a broad of frequencies, due to the brine motion resulting in this patchy saturation zone.

In addition to frequency dependent velocities (as shown in Figure 4.10), a frequency dependent reflection coefficient can be calculated. Figure 4.11 shows the reflection coefficient for a frequency of 10 Hz (seismic) and for 10000 Hz (logging). The difference plotted between these two reflection series is also shown in Figure 4.11.

Dispersion and attenuation can be related with the used Cole-Cole relation, and as a result, the loss of energy as function of frequency can be described (Figure 4.12). A larger dispersion will lead to higher attenuation within the sediments. The highest attenuation, better $1/Q$, is predicted at the gas-brine interface at the upper and lower sand, while the zones with 100% brine and high gas concentrations (90% gas saturation in the upper sand) indicate less attenuation.
Figure 4.10. P-wave Modulus derived from the GOM-1 well log (Figure 4.9) at seismic frequencies (Seismic 10 Hz - red) and logging frequencies (Well Log 10000 Hz - black)
Figure 4.11. P-wave reflection coefficient (RC) from the GOM-1 well log (Figure 4.9) at seismic frequencies (Seismic 10 Hz - solid line) and logging frequencies (Well Log 10000 Hz - dotted line). High attenuation occurs at the gas-brine interfaces.
Figure 4.12. Inverse P-wave quality factor (1/Q_p) derived from the GOM-1 well log (Figure 4.9) at seismic frequencies (Seismic 10 Hz - solid line) and logging frequencies (Well Log 10000 Hz - dotted line). High attenuation occurs at the gas-brine interfaces.
4.4 Discussion and Conclusions

We developed a predictive method to derive frequency dependent reflection and attenuation logs. This procedure is based on assumed mechanisms and includes frequency-scale dependence. We showed that by using the limiting moduli ($M_\infty$ and $M_0$), the relaxation time ($\tau$), and the distribution parameter ($\kappa$) at any point in depth, a velocity can be derived for wide range of frequencies. By using the four parameters of the Cole-Cole equation, we can predict velocities and attenuation at any frequency, which can be used to produce frequency dependent synthetic seismograms.

Differences between the reflection and attenuation logs for the seismic and well logging frequencies are highest for partial gas saturated rocks with high brine saturation, and across the interface between gas and brine filled rocks. No large differences in the P-wave velocities exist in 100% brine saturated rocks and gas saturated rocks, which also results in small attenuation within the investigated frequency range, seismic to sonic frequencies.
Chapter 5

CARBON SEQUESTRATION TIME LAPSE SIGNATURES

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Geophysical techniques have been used to monitor a carbon dioxide (CO<sub>2</sub>) sequestration project in the Utsira formation of the North Sea. The Utsira formation is a shallow saline aquifer at a depth of less than 1000 m. This formation is a good target for field studies due to its high porosity and permeability. Waste CO<sub>2</sub> from the processing of the deeper Sleipner formation gas is injected into the Utsira. Seismic imaging, among other methods, has been very successful in tracing the CO<sub>2</sub> plume in the subsurface and understanding the distribution and movement of the CO<sub>2</sub>. Previous studies have not detected any vertical migration of the CO<sub>2</sub> through the shale above the Utsira formation.

The ability to quantitatively ascertain the amount of CO<sub>2</sub> distributed in the Utsira formation is a new focus of research efforts. With recent increase in data quality, quantitative interpretation, especially for time lapse studies, is feasible. Conventional
Gassmann fluid substitution does not provide the correct model for the observed seismic response. Here we propose a new description of the seismic response of a very permeable reservoir to predict the interaction of the $CO_2$ and the brine in the reservoir by using a new frequency-dependent velocity model for this partially saturated reservoir.

Overall, seismic monitoring is a small part of the effort to understand and deal with the challenges of greenhouse gases and global warming. The Norwegian government has provided tax incentives, which made it possible to develop a technology to sequester $CO_2$ in a deep saline aquifer in the North Sea. This paper will put seismic monitoring into context of the overall goal of carbon sequestration.
5.1 Introduction and Background

The challenges of global climate change are highly controversial. Increasing average temperatures have been observed, as well as tide and ecological changes. However, the extent of these changes due to humankind is difficult to determine. The impact from the social environment to the economy is even more difficult to assess. Despite this, we do not have the time to wait and see what will happen. Large potential exists in the earth sciences to assess and evaluate the global challenges. One example is remote sensing, which can be used to monitor the change of vegetation; this can indicate change of the climate. As a first response, CO$_2$ sequestration schemes and projects have been developed to sequester CO$_2$ through geological, terrestrial, and oceanic techniques.

Geophysics is a part of this development to find and characterize sites for CO$_2$ storage in saline aquifers or in oil reservoirs for Enhanced Oil Recovery (EOR), or to monitor its safe storage. The proposed model could explain the differences of CO$_2$ volume calculations. Another advance in the understanding of the time lapse response would be the acquisition of multi-component/ocean bottom cable seismic surveys. While the seismic response of conventional - P-wave data is largely affected by gas/CO$_2$ saturation, converted wave data can resolve the problems of imaging below the CO$_2$ plume. The Sleipner West project has shown that geophysical methods can be used to monitor a CO$_2$ sequestration in a saline aquifer. No breach of the seal has been noted and the movement of the plume has been outlined with great detail. However, this assessment accounts only for the first five years of a 25-year injection plan. Additionally, the total lifetime of CO$_2$ in the aquifer will be much longer as CO$_2$ dissolves in the formation brine. Continuous monitoring could provide an improved understanding of the distribution, migration, and long-term effects of
$CO_2$ in the saline aquifer.

The Sleipner field produces natural gas with a high content of $CO_2$. The limit for the amount of $CO_2$ in exported gas is 2.5%. However, the gas produced from Sleipner has a $CO_2$ content of 9.5% (Korbol & Kaddour, 1996). Since Norway taxes $CO_2$ emissions at 320 NOK/t (40 USD/t), releasing the $CO_2$ to the atmosphere is an expensive option for its disposal. To avoid taxation and gain technical expertise in carbon sequestration, Statoil decided to start a project at Sleipner to sequester $CO_2$. Torp & Brown (2004) discuss the costs of the injection of $CO_2$ in the saline aquifer above the Sleipner gas field. The initial costs to drill and complete the injection well are 120 MNOK (15 MUSD). In comparison, one repeated 3D seismic data acquisition costs 3.2 MNOK (0.4 MUSD). Annually, 1 million tons of $CO_2$ will be injected over 25 years, at an operational cost of 54 MNOK (6.8 MUSD) per year.

At Sleipner, after $CO_2$ is separated from methane, it is injected into the Utsira formation, a saline aquifer above the original gas reservoir. The saline aquifer of the Utsira formation, which is a Miocene-Pliocene sandstone (Gregersen et al., 1997) with high porosities 35-40% and a thickness up to 300 m (Zweigel, 2000). One aspect that makes this aquifer a good candidate for sequestration is the Nordland Shale layer with a thickness of 250 m above the Utsira formation. Up to the present, three seismic data volumes have been acquired. One seismic data acquisition was done before the start of the injection in 1994. After the injection started in 1996, two more seismic acquisitions were done in 1999 and 2001. Figure 5.1 shows the injected volume of $CO_2$ into the Utsira formation and the well head pressure of the injector well. The injected volume is equivalent to 1 million tonnes of $CO_2$ per year. No increase in the well head pressure can be observed, which indicates that the reservoir can be modeled as to be of infinite volume with very high permeability.
Figure 5.1. Top: Injected volume of $CO_2$. The start of injection was 1996, the base survey was collected in 1994. Bottom: Well head pressure in MPa of injector well. The start of injection was 1996. Decreases in pressure indicate shutdown of the injection due to maintenance.
5.2 Reservoir Monitoring of $CO_2$ at Sleipner

5.2.1 Seismic Response at Sleipner and Reservoir Models

$CO_2$ is injected at 6 MPa well head pressure, which results in a reservoir pressure of nearly 10 MPa. Under this pressure and at $37^\circ C$ reservoir temperature at the injection point, the gas is in supercritical condition (neither a gas nor a liquid), which makes the determination of the properties much more difficult. Due to the density differences between brine (1g/cc) and $CO_2$ (0.4g/cc), and if the permeability of the formation is homogeneous and continuous, all $CO_2$ would accumulate at the interface between Utsira sand and the Nordland shale and form a gas (supercritical $CO_2$) cap. Lindberg et al. (2000) modeled the distribution of the $CO_2$ in a homogeneous reservoir, as shown in Figure 5.2.

The first reservoir models, based on the information from the seismic data from 1994, illustrate an apparent homogeneity of the Utsira formation. Shale layers which are sub-seismic resolution within the Utsira formation have not been identified in the base survey 1994, Figure 5.3, and assumed not to be of importance. Overall, the seismic properties of the shale and brine saturated sands are very similar, and the impedance contrast is very small. As a result it was not possible to identify these shale layers by seismic imaging.

Shale layers have been observed and identified in well logs by using gamma ray tools. From these tools, the thickness of the shale layers can be measured. These shale layers are thin only a few meters thick and cannot be correlated between well locations, which leads to the conclusion that they are discontinuous.

The value of the seismic data in the reservoir monitoring process has been shown by Eiken et al. (2001), see Figure 5.3. Imaging the saline aquifer after the sequestration process indicated that the model of a homogeneous reservoir could not be
Figure 5.2. $CO_2$ saturation cross section map after 861 days; blue indicates brine saturation, red indicates $CO_2$ saturation; from Lindberg et al. (2000)
supported, see Figure 5.4. Several barriers for the $CO_2$ migration can be observed from the time lapse response. The time lapse response highlights the effect of the $CO_2$ collecting under the shale layers.

A geological model, Figure 5.5, was proposed in earlier works of Zweigel (2000). This model forecasted the $CO_2$ migration will be affected by intra shale layers. Incorporation of the stepped migration paths into the reservoir model gives a more accurate model (Eiken et al., 2001). Five major barriers have been interpreted from the time lapse response, which cause an inhomogeneous distribution of $CO_2$ in the Utsira formation.

Eiken and others then attempted to calculate the volume and mass of the $CO_2$ in the reservoir: Eiken et al. (2000), Eiken et al. (2001), Arts et al. (2002) and Eiken (2003). The seismic time lapse signatures between 1994 and 2001 are caused by the differences in the acoustic properties of the rock saturated with brine and with $CO_2$. The properties of $CO_2$, $CO_2 +$ methane ($CH_4$), and brine are plotted in Figure 5.6. Brine is 10 times less compressible than $CO_2$. Including 10% of methane to $CO_2$ does not change the properties significantly. Wood’s equation (Wood, 1955) was used to calculate the mixture properties of carbon dioxide and brine. The compressibility of the fluid mixture is dominated by the high compressibility of $CO_2$.

Finally, the fluid properties of the $CO_2$-brine mixture are used for Gassmann’s equation (Gassmann, 1951) to predict the change in the rock compressibility if $CO_2$ is injected. As a result of the significant differences in the fluid properties, the rock saturated with $CO_2$ has a much lower modulus than if it was saturated with brine. This comparison is shown in Figure 5.7.
Figure 5.3. Base-line seismic survey - 1994 - top and base of the Utsira Formation are indicated
Figure 5.4. Repeated seismic survey - 2001 - top and base of the Utsira Formation are indicated.
Figure 5.5. Geological model of the $CO_2$ migration process (Zweigel, 2000). The Utsira sand is surrounded by the Nordland shale on the top and the Hordaland shale on the bottom.
5.2.2 Volume Calculations

The differences in the seismic properties between brine and $CO_2$ can be used to extract more information from the time lapse seismic response. Two conceptual paths have been used to extract the amount (volume and mass) of $CO_2$, which was injected. The first method uses the strength of a seismic reflector to estimate the thickness of the $CO_2$ layer under the shale barriers. The lateral extension of the $CO_2$ plume was mapped for the different levels of $CO_2$. Next, the change of the amplitude with $CO_2$ thickness under the shale layer can be modeled. If the thickness of the $CO_2$ layer increases, the amplitude of the seismic reflections also increases, up to it reaches the thickness of a quarter of the wavelength; this is also known as tuning thickness. The volume of the reservoir occupied by the $CO_2$ was obtained from this measurement. To calculate the total volume of $CO_2$, the $CO_2$ saturation under the shale layers has to be known. Zweigel (2000) reports the capillary pressure curve of the sand with a brine $CO_2$ mix from core measurements. Due to the high porosity,
Figure 5.7. Properties of rock saturated with $CO_2$ and brine as a function of $CO_2$ saturation.
the $CO_2$ saturation in the sand layer under the shale barriers can be assumed to be at 90% saturation. To calculate the mass of injected $CO_2$, the volume had to be multiplied with the density of the $CO_2$. Since the injection pressure and temperature are known, the density of $CO_2$ can been calculated.

The second method used to calculate the volume and mass of the $CO_2$ in the Utsira formation is to measure the push down effect of reflectors below the Utsira formation. This method uses the response of time lapse seismic surveys directly. While $CO_2$ is injected into the reservoir, the velocity in the sandstone filled with $CO_2$ drops to a lower velocity compared to the same sandstone saturated with brine. As a result, the travel time of a reflector under the $CO_2$ filled formation would be larger, in comparison with a formation that is brine filled. If the travel time differences and the rock properties are known, the cumulative sum of the $CO_2$ layer thicknesses can be calculated. Arts et al. (2003) presented the results of this method.

Both techniques used a simple Gassmann calculation to estimate the amount of $CO_2$ in the formation; the method using the reflection strength underestimated the volume of $CO_2$ (the calculated volume is $4.7 \cdot 10^6 \text{ m}^3$, while the injected volume was $12 \cdot 10^6 \text{ m}^3$); using the time delay method overestimated the mass of $CO_2$ (the travel time delay measure over the area is $9900 - 11200 \text{ m}^2\text{s}$, while thin layers of $CO_2$ only predict a time delay of $3800 \text{ m}^2\text{s}$). One possible reason for the differences can be explained by the over-simplified model used to predict the velocity changes with $CO_2$ saturation in the case of the Utsira formation.

### 5.2.3 New Velocity Model for the Utsira Sand

Both methods to calculate the volume and mass of $CO_2$ showed large differences, which cannot be explained by simple variations in density or reservoir heterogeneity.
Here we develop a new model for the velocity distribution after $CO_2$ was injected. Arts et al. (2003) produced several different scenarios for the $CO_2$ saturation in the Utsira formation, as shown in Figure 5.8. Different distributions of $CO_2$ can be modeled for a given volume of $CO_2$. For one end member of the $CO_2$ distribution, the $CO_2$ is uniformly distributed in the reservoir. In this case the saturation of $CO_2$ is 8% and the brine saturation is 92%. The modeled time delay is very large (nearly 300%) compared to the measured time delay from the acquired seismic data. The other end member of the $CO_2$ distribution is that all the $CO_2$ is distributed in thin layers of $CO_2$ (100% $CO_2$, 0% brine) under the shale layers.

Using this model, the time delay is under predicted by 60%. All the other models have $CO_2$ distributions in the formation between these two end members. However, the measured capillary pressure curve (Lindberg et al., 2000) shown in Figure 5.9, can restrict the models. Since the rock has a high porosity and permeability, the capillary pressures, as function of water saturation, is very small. This results in high $CO_2$ saturation under the shale barriers with a sharp boundary between the $CO_2$ and the fully brine saturated zones.

Gassmann’s equation (Gassmann, 1951) is a common tool to predict these changes in velocities for different fluid saturations. However, it is not always clear what the fluid distribution and boundary conditions are. A saturation distribution profile derived from a well log can be used to calculate the elastic properties of the section for a specific frequency. A scaling of the frequency response has to be done to be able to develop a model, e.g., the seismic response.

A schematic drawing to better understand the challenges are shown in Figure 5.10. A rock saturated with $CO_2$ overlays the same rock saturated with brine, so both zones are connected. For measurements at very high frequencies, the different pore
Figure 5.8. Properties of rock saturated with $CO_2$ and brine as function of $CO_2$ saturation
Figure 5.9. Properties of rock saturated with $CO_2$ and brine as function of $CO_2$ saturation
fluids are mapped extensively. The rock saturated with \( CO_2 \) has a much lower velocity compared with the rock saturated with brine (Figure 5.6). On the other side, at low frequencies, the whole section appears partially saturated. Since the different zones are interconnected with no boundary between them, the effective fluid modulus can be calculated using Wood’s equation (1955), which is dominated by the compressibility of the \( CO_2 \) (Figure 5.10). White (1975) proposed a mechanism for attenuation and velocity dispersion of seismic waves in fluid saturated rock in the presence of gas pockets. A fluid motion caused by unequilibrated pore pressures causes attenuation and dispersion. Similarly, Gist (1994) developed a model to explain experimental data for partially saturated rocks.

This mechanism can be modeled by accounting for a pressure diffusion caused by a seismic wave. Starting with the one-dimensional diffusion equation for pressure, \( P \):

\[
\frac{\partial^2 P}{\partial x^2} = \frac{1}{D} \frac{\partial P}{\partial t}, \tag{5.1}
\]

with a diffusion coefficient \( D \) of:

\[
D = \frac{k}{\eta \phi} K_{fl}. \tag{5.2}
\]

This diffusion coefficient depends on the fluid mobility, permeability \( k \) and viscosity \( \eta \), the porosity \( \phi \) and the incompressibility of the fluid \( K_{fl} \). The solution of the diffusion equation can be written in the following form (Mavko \textit{et al.}, 1998):

\[
P(x, t) = \frac{P_0}{\sqrt{4\pi Dt}} e^{-x^2/4Dt} = \frac{P_0}{\sqrt{4\pi Dt}} e^{-\tau/t}. \tag{5.3}
\]

A relaxation time \( \tau \) of a pressure pulse will depend on the distance \( x \) the fluid has to
Figure 5.10. Schematic understanding of fluid distribution and rock properties - CO$_2$ saturated rock on top of brine saturated rock. Depending on the measurement frequency, different velocity profiles can be predicted. Left, for very high frequencies, the velocity profile will map the saturation differences; CO$_2$ is highly compressible compared with brine, which results in low velocities compared to the lower, brine saturated rock. On the right side of the figure, the velocity profile is plotted for very low frequencies (static conditions), since the brine and CO$_2$ saturated rock are connected the whole zone appears partially saturated (Wood’s equation (1955)).
Figure 5.11. Diffusion length as a function of seismic frequencies for the properties of the Utsira formation

equilibrare over, and is also dependent on the diffusion coefficient $D$:

$$\tau = \frac{x^2}{4D}. \quad (5.4)$$

Using the properties known for the Utsira formation, a frequency dependent diffusion length can be calculated as shown in Figure 5.11. It is demonstrated that the diffusion length decreases as the frequency increases. As a result, the reflection coefficient is frequency dependent.

In this case of $CO_2$ sequestration, $CO_2$ is accumulated under numerous shale barriers, and under each $CO_2$ layer, the rock is 100% brine saturated. At very low frequencies, the diffusion length is long. This means that the distance over which fluid
Pressure can come into equilibration is also very long. Within this diffusion length, the boundary conditions are effectively open. As a result, Gassmann’s equation cannot be applied directly. At the CO₂ - brine interface, the brine pressure can equilibrate. This has the effect that the rock frame would be drained. However, since the density of brine is greater than the density of CO₂, the velocity of the brine saturated sand would be lower than the velocity of the CO₂ saturated rock. At distances larger than the diffusion length, the brine saturated formation can be described as having closed boundary conditions. Here, the velocity is determined by the undrained conditions, also known as Gassmann’s equation.

The velocity profile for several diffusion lengths (= several distinct frequencies) has been calculated for the simple case of a 2 m CO₂ layer below a 2 m shale barrier. For each frequency, the diffusion length has been calculated and the velocities between the interface and the brine have been linearly interpolated. After the P-velocity profile has been calculated, the S-wave velocity and density profiles can be derived. In contrast to the P-wave profile, the S-wave and density profiles have not been affected by the diffusion process.

The velocity profiles are then convolved with a Ricker wavelet with bands corresponding to the diffusion calculations mentioned above. For example, the velocity profile for 20Hz has been convolved with a Ricker wavelet centered at 20 Hz (Figure 5.12). Differences between the 80 Hz trace and the 10 Hz trace can be observed by looking at the brine shale interface at 290 ms. The push down effect is greater for the 10 Hz trace, due to the greater relative change in the velocity profile.

In general, the velocity profiles shown in Figure 5.12 and the synthetic traces are over-simplifications of the geological model. The gamma ray log of the injection well was used to create a more representative model (Figure 5.5). To create the impedance
Figure 5.12. Velocity profile (color bar: blue low velocities (sand) and brown high velocities (shale)) and synthetic seismograms for no diffusion, 80 Hz, 40 Hz, 20 Hz, and 10 Hz (left to right).
Figure 5.13. P-wave velocity profile for different velocity models. From left to right: velocity profile before injection, profile for 2 m CO$_2$ layer under shale, profile for 2m CO$_2$ layer at 10 Hz, including diffusion process, 20 Hz, 40 Hz, 80 Hz trends, several wells from the area have been used to calibrate the P-wave, S-wave and density values for the brine saturated sand and shales. From the gamma ray log, eight shale layers have been identified within the Utsira formation, ranging between 1-2 meters. Figure 5.13 shows the P-wave velocity profile before and after the injection. If the diffusion process is included in the modeling, large differences can be observed comparing the profiles between 10 Hz and 80 Hz with the velocity profile using a normal Gassmann model. S-wave velocity and density profiles are not affected by the diffusion process, so they show no frequency effect.

Figure 5.15 shows the velocity profiles and the synthetic traces for Ricker wavelet of frequencies of 10 Hz, 20 Hz, 40 Hz, and 80 Hz, before the injection of CO$_2$ into
Figure 5.14. S-wave velocity (left) and density (right) profile for different velocity models. From left to right: velocity profile before injection, profile for 2 m CO$_2$ layer under shale, profile for 2 m CO$_2$ layer at 10 Hz, including diffusion process, 20 Hz, 40 Hz, 80 Hz

the reservoir. Since the sequence 100% brine saturated sands and shales, the velocity profiles are independent of frequency; no macroscopic fluid flow is influencing the velocity profile. In this case, the overall response for the Ricker wavelet of 10 Hz is very different from the response for 80 Hz. With increased frequency, smaller scale heterogeneities/features can be resolved.

While only the top and bottom reflection of the Utsira formation can be identified at the 10 Hz trace, thicker shale layers can be identified looking at the synthetic trace for 80 Hz.

Using the modeled fluid distribution, a 2 m thick CO$_2$ layer was put under each of the shale layers. High porosity and permeability correspond with low capillary pressures; under these conditions the CO$_2$ saturation of the sand can be assumed to be 90% (Arts et al., 2002). The increase in saturation of CO$_2$ will decrease the P-wave velocities in the sand, while the S-wave velocity will not change significantly. The response for a conventional velocity distribution is shown in Figure 5.16.
Figure 5.15. Seismic model of Sleipner reservoir before $CO_2$ injection for Ricker wavelets with a frequency of 10 Hz to 80 Hz. Colors indicate the velocity profile, shales have the highest velocities (dark brown), $CO_2$ saturated sands have the lowest velocities (dark blue).
Figure 5.16. Seismic model of Sleipner reservoir after $CO_2$ injection for Ricker wavelets with frequencies of 10Hz to 80Hz using a conventional distribution of $CO_2$ under the shale layers. Colors indicate the velocity profile, shales have the highest velocities (dark brown), $CO_2$ saturated sands have the lowest velocities (dark blue).
Here the velocity profile is independent of frequency. Again, for lowest frequencies, only the top and bottom of the reservoir can be distinguished. If the frequencies of the wavelet increase, more and more features of the reservoir can be imaged. For the highest frequencies, the shale layers with the $CO_2$ distributed under them can be imaged. Differences in polarity for the bottom reflection of the reservoir, at 970 ms, can be observed by examining the synthetic trace for 80 Hz. While a positive polarization can be observed for 10 Hz, 20 Hz, and 40 Hz, the polarity for 80 Hz seems to be negative. This is an interface effect.

One explanation for the differences in the polarity for the response at 80 Hz is shown in Figure 5.17. This figure shows the synthetic seismic traces using the conventional velocity saturation and convolved with a Ricker wavelet at 80 Hz. One trace was created excluding all the multiple reflections and mode conversions, while the second trace was created including multiples and conversions. It can be seen that the reflection at the bottom of the reservoir cannot be identified if the multiples are included in the modeling of the seismic trace. The negative amplitude for the synthetic, including all multiples and mode conversions, causes interferences with the reflection of the bottom of the Utsira sand at this frequency. As a result, it will be very difficult to identify and measure the time delay between two seismic acquisitions. A strong reflector at greater depths should to be chosen to measure the time shift between surveys.

Figure 5.18 shows the velocity profile and the synthetic trace (including multiples and conversions) for frequencies from 10-80 Hz, where the diffusion length within the high porosity sand is frequency dependent. As a result, the effective thickness of the low velocity zone under the shale layer is also frequency dependent. At 10 Hz, the diffusion length is almost 10 times longer when compared to 80 Hz. Consequently,
Figure 5.17. Seismic model of Sleipner reservoir after $CO_2$ injection for a Ricker wavelet of 80Hz using a conventional distribution of $CO_2$ under the shale layers. Left trace was modeled excluding all multiples and mode conversions, while the middle trace was modeled accounting for multiples and mode conversions. Right trace is the difference between the traces (left and middle). A significant difference can be seen at the bottom reflector at 970 ms.
Figure 5.18. Seismic model of Sleipner reservoir after CO$_2$ injection for Ricker wavelets with frequencies of 10-80 Hz using a velocity distribution of CO$_2$ under the shale layers accounting for macro flow mechanism. Depending on the dominant frequency, the velocity profile is changing. Colors indicate the velocity profile: shales have the highest velocities (dark brown), CO$_2$ saturated sands have the lowest velocities (dark blue).

The time of the bottom reflection increases from 970 ms for 80 Hz to 1000 ms for 10 Hz. For low frequencies, again the top and bottom of the reservoir can be identified. At high frequencies multiples and mode conversions interfere with the signature of the bottom of the reservoir.

Recorded seismic traces are not only the reflection for a narrow frequency band; the seismic response is a superposition of responses for a range of frequencies. To model the seismic signature of the Utsira formation, the synthetic traces for each
frequency $f$ are stacked to represent the amplitude $A(t)$ response of the true survey:

$$A(t) = \int x(f) A'(t,f) \, df \approx \sum_i A'(t, f_i). \quad (5.5)$$

Not all the frequencies should be weighted equally in the stacked response to represent the bandwidth of the Sleipner surveys. The power spectrum of the seismic response around the target interval is plotted in Figure 5.19. As a result, the seismic response for different frequencies has to be stacked using different weights, represented as $x$ in Equation 5.5 (Table 5.1).

Figure 5.20 shows the final result of the frequency stacked traces, applying the weighting factors in Table 5.1. All three traces show significant differences. Again, only the top and bottom of the Utsira formation can be identified for the model
Table 5.1. Weighting factor for synthetic traces for different frequencies

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.35</td>
</tr>
<tr>
<td>40</td>
<td>0.35</td>
</tr>
<tr>
<td>80</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Figure 5.20. Frequency stack of the modeled traces for the Sleipner reservoir. Left trace is the frequency stack of the response before $CO_2$ injection, while the middle trace is the stack response after $CO_2$ injection for conventional distribution. The right trace shows the frequency stacked response for the modified velocity profile accounting the macroscopic fluid flow.
Figure 5.21. Modeled time-lapse response for the frequency stacked seismic responses. Left trace shows the frequency stack modeled response applying the macro flow model minus the model response before CO₂ injection, while the middle trace shows the stack response for the conventional distribution minus the response before CO₂ injection. The right trace shows the difference between the modeled trace using the reflection series for the macro flow and conventional velocity distribution.

without CO₂ saturation. Significant reflections can be identified in the model with sand, while the conventional fluid distribution model negative amplitudes can be correlated with the presence of the CO₂. Looking at the stacked frequency response using the diffusion / macro flow model, it can be observed that it is more difficult to correlate between the seismic amplitude and layers of CO₂. However, it is important to note that reflection of the bottom reflector occurs at a later time, compared to the conventional model.

The difference between the two models and the seismic response before the CO₂
injection are shown in Figure 5.21; this can be seen as a time lapse signature. Trace a) shows the difference between the diffusion model and before the injection; trace b) shows the difference between the conventional model and before CO$_2$ injection. The last trace shows the differences between the two modeled CO$_2$ distributions. Significant differences can be seen in both models, especially in the center of the trace. Next, the model should be verified by the data from the CO$_2$ injection project. Several methods can be used to calculate time shift.

First, a reflector can be selected before and after the injection. Second, the cross correlation between the two seismic data volumes on a target reservoir can be used to define the time shift between pre and post injection. Dumont et al. (2001) used the slope of the power spectrum of the cross correlation to extract the time shift. This technique was tried but showed random results; discussions with Gret (2004) showed that this method is unstable. In contrast to the model (Figure 5.15) in the acquired seismic data (Figure 5.22), no significant interface below the Utsira formation was available to make accurate push down measurements; the traces have been limited to 2 sec recording lengths. One reflector at 1330 ms was used to investigate its potential for measuring time shifts. It can be seen that the reflection strength for this reflector at 1330 ms is influenced by the anomaly caused by the injected CO$_2$.

Another challenge is to measure the time shift as a function of frequency. If the proposed theory is right, then a greater time shift should be observed at lower frequencies and a smaller time shift at higher frequencies. At first, the idea was to use near and far stacked offset cubes, speculating the near offset cube would have a higher frequency content compared to the far offset cube. Comparison between the amplitude spectra between 1200 and 1400 ms shows no significant differences. Next, the near offset traces have been filtered for high and low frequencies. The resulting
Figure 5.22. Seismic line before (1994) and after (2001) injection. The reflector at 1330ms is difficult to trace after the injection.
spectra of the band pass filtered data are shown in Figure 5.23.

Now, the cross correlation can be used between the traces from the pre (1994) and post (2001) injection seismic acquisitions at the target reflector below the Utsira at 1300 ms. The results can be seen in Figures 5.24 and 5.25. Figure 5.24 shows the amplitude of the cross correlation between pre and post injection. For these calculations only a small part of the seismic data is used; one line with traces from 1850 to 1900 (compare with Figure 5.22). Only near the edge of the injected area was a reflector identified at 1300 ms below the injection point and the CO$_2$ plume. The amplitude of the cross correlation, Figure 5.24 indicates that the results for the time shift estimation of the cross correlation can be used. For traces outside of the area affected by the CO$_2$ the amplitude is approximately one, while the amplitude will decrease for traces inside the area with the CO$_2$.

The results in Figure 5.25 show the differences between time shifts for high frequencies (HP) and low frequencies (LP). For traces outside of the region with the CO$_2$ plume (higher than 1890), the time shift is zero for both frequency ranges.
Figure 5.24. Amplitude of the cross correlation for the low (LP) and high (HP) frequency filtered seismic traces.
Figure 5.25. Time shifts calculated for the low (LP) and high (HP) frequency filtered seismic traces
The time shift increases with the decrease of the trace number for decreasing trace numbers from 1890-1875. This effect can be explained by the increase of $CO_2$, as long as the traces are closer to the injection point. While only one $CO_2$ layer is affecting the seismic response for traces on the outside of the plume, in the middle of the $CO_2$ plume a stack of $CO_2$ layers is affecting the seismic response.

As a result, the recorded seismic signal is highly contaminated with interferences of within the $CO_2$ plume for traces smaller than 1875. A different time shift between the cross correlation for high frequencies and low frequencies can be observed in the data (see Figure 5.25). If the frequency band of the seismic data is higher, time shifts are smaller. This observation is consistent with the proposed model of a macroscopic diffusion process for this higher permeability sand. Overall, a better analysis can be done if a stronger reflector could be identified below time lapse anomalies. The reflection strength of the reflector at 1300 ms is not strong enough to make a more detailed study of the time lapse response.
5.3 Conclusions

Sleipner has been a very successful example of time lapse seismic applications to image the migration of injected CO$_2$ in a saline aquifer. Migration paths have been observed, which have been dominated by the heterogeneities in the Utsira formation. Thin, discontinuous shale layers prevent the CO$_2$ from moving straight upwards. Under these permeability barriers, some of the CO$_2$ is collected and forms layers.

Since the anomalies have been so clearly observable, the next step was to try to back-calculate the amount of CO$_2$ injected into the Utsira formation. Several techniques to calculate the right volume have been proposed, but partially failed. One challenge in using seismic data to extract CO$_2$ volumes is how to apply the right model to predict changes in velocities caused by the injected CO$_2$; simply applying Gassmann’s equation will lead to incorrect results. We have proposed a new methodology to predict the seismic properties in the reservoir, including the effect of pore pressure diffusion at the CO$_2$ - brine interface.

Applying this new model to the case of CO$_2$ sequestration at Sleipner results in frequency dependent velocities and reflection coefficients, as well as frequency dependent time shifts, which were noted in the first test. Since the reflector which was used to make these time-shift measurements was not very strong, the results should be confirmed with further testing. For the future, a reflector located below the Utsira formation should be considered and these frequency dependent time-shift measurements should be repeated, including the anomaly.
Chapter 6

CONCLUDING REMARKS

We have seen that rock properties may have significant frequency dependency. This frequency dependency can be observed by dispersion or attenuation measurements. These measurements, especially when done at low frequencies, lead us to question some commonly applied rock physics models. Not that these models are wrong; however, significant violations of the boundary conditions or assumptions can result in incorrect predictions. Large attenuation and dispersion can be observed if large contrasts in elastic properties are present. Inhomogeneities in saturation or in the rock matrix lead to large attenuation and dispersion effects.

To our surprise, elastic properties of shales have shown to be strongly frequency dependent in the case of high porosity shales. A low porosity shale sample showed no significant dispersion. However, we have to differentiate between responses of the shear and normal stiffness. While shear stiffness showed no significant dispersion, the normal stiffness showed significant differences: over 50% between the low frequency measurements and ultrasonic measurements. This trend has been verified by other measurement techniques, including VSP, sonic-log, and micro indentation.

More measurements should be done, especially for shales, to better understand the dispersion mechanisms and the cause. Since the mineralogy of both measured shales is very similar and only the amount of pore space is different, the first hypothesis would be the pore space is the main factor for this observed dispersion. The next step is to extract attenuation data for these shales to understand the correlation
between attenuation and velocity dispersion. The assessment of attenuation of shales can have significant implications for seismic imaging, like migration and amplitude extraction and interpretation. A just argument is that most of the transmission of seismic waves is through shales and will affect wave propagation significantly.

Seismic reflections are caused by impedance differences of interfaces. To understand what effect dispersion has on these reflections, the results from the dispersion measurements in sandstones with different pore fluids and the results for the high porosity shale, shown in Figure 6.1, are combined, and a frequency and offset dependent reflection coefficient (RC) (Figure 6.2) is calculated. In the case of the brine saturated sandstone, the dispersion of the sandstone is small, but the dispersion of the shale is high; thus we can observe a significant frequency dependency on the reflection coefficient. On the other hand, for the butane saturated sandstone, both rocks behave similarly, and as a result, the reflection coefficient shows only a small dependency on frequency.
Figure 6.2. Reflection coefficient (RC) as function of frequency and offset for a shale (top) - sandstone (bottom) interface. The sandstone is brine saturated (left) and 30% butane and brine saturated (right), properties from Figure 6.1.

Two different cases of implementing the low frequency results have been shown, one using well log data to predict the seismic frequency response. We have frequency scaled the information from the well logs and predicted an reflection series as a function of frequency as well as the attenuation for each frequency, including for seismic frequencies. The second example was to improve the understanding of the time lapse results for the CO$_2$ sequestration project at Sleipner. Inhomogeneities in the form of CO$_2$ layers under shale breaks can substantially influence the derived velocity models of the reservoir.

We only scaled the frequency response of well logs and predicted the seismic response, so no geometric upscaling has been done to predict the influence of heterogeneity on the seismic response. Further research can extend the scope of this work substantially. Much more work has to be done to couple both frequency and geometric scaling and to determine what “seismic wiggles” really mean.
REFERENCES


Arts, R.J., Chadwick, A., Eiken, O., & Zweigel, P. 2003. Interpretation of the 1999 and 2001 time-lapse seismic data. TNO.


